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New Ranking Function Technique for Fully Fuzzy Linear Programming Problems Utilizing Generalized Decagonal Membership Function

Israa Hadi Hasan[1], Iden Hassan Al-kanani[2]

[1]Department of Mathematics
College of Science
University of Baghdad
Baghdad, Iraq

[2]Department of Mathematics
College of Science for Women
University of Baghdad
Baghdad, Iraq

email: Israa.H.Hasan@uotechnology.edu.iq,
idanha_math@csw.uobaghdad.edu.iq

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Abstract

The fuzziness approach is useful when trying to solve programming problems with a degree of ambiguity. In recent years, various approaches to the difficulties of fuzzy linear programming have developed. Many papers use the features of fuzzy sets as a method to find the best solutions to fuzzy programming problems, and some of these papers deal with fully fuzzy linear programming problems in which all variables are triangular fuzzy numbers or trapezoidal fuzzy numbers. For the fully fuzzy linear programming (FFLP) problems, where all the variables and parameters are of the same type as decagonal fuzzy numbers, this study provides a new ranking function technique

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and a new membership function of decagonal fuzzy numbers. Using the arithmetic decagonal fuzzy numbers and the proposed ranking function technique, a new algorithm for a fully fuzzy simplex (FFS) method is offered to obtain the optimal fuzzy solution. In the paper's applied section, we provide a working example of how to establish a fuzzy optimal solution to the given problem.

1 Introduction

One of the most often used methods in operations research is linear programming. The fuzziness approach is useful when trying to solve programming problems with a degree of ambiguity. The fuzzy programming technique may be beneficial when the coefficients are uncertain numbers or quality. Many papers use the features of fuzzy sets as a method to find the best solutions to fuzzy programming problems. Zimmermann [1] first proposed the concept of Fuzzy Linear Programming (1978). In 2010, Kumar and Kaur [2] came up with a ranking technique and proposed a generalized simplex algorithm, both of which can be used to solve a specific class of fuzzy linear programming (FLP) problems. In 2013, Hassan and Saeed [3] introduced a new method that crosses the divide between both types of linear programming problems, one involving fuzzy specialized coefficient numbers and the other involving parametric modifications to the objective function's coefficients. In 2014, Rajarajeswari and Sudha [4] proposed an innovative approach for solving the FFLP problem by utilizing a ranking algorithm. The supplied FFLP problem is transformed into a crisp linear programming problem with bound variable restrictions to find the optimal solution. In particular, Hassan and Alaa [5] focused on linear programming problems with fuzzy objective function coefficients, fuzzy right-hand side coefficients, or both. Then, various linear ranking functions were used to find a solution for these fuzzy linear programming problems. In 2018, the ranking of generalized octagonal fuzzy numbers was proposed by Ghadle and Ingle [6]. The suggested method employs a Euclidean distance-based in the center of centroids. They used an alternative simplex method to get the optimal solution to the generalized octagonal fuzzy linear programming problem. In 2022, the trapezoidal function, a linear membership function, was used alongside fuzzy numbers. Fuzzy network models were treated using the Yager ranking algorithm of Ghadle and Ingle [7]. In this paper, we deal with the fully fuzzy linear programming problem where the coefficient of the objective function and the constraints depend on generalized decagonal fuzzy numbers. Using the arithmetic of generalized decagonal fuzzy numbers operations, employing the new ranking function technique, and developing a generalized algorithm of the fully fuzzy simplex method the optimal fuzzy solution of the (FFLP) problem is reached, the given example shows the steps of the solution. This paper is organized in nine sections. In Section 2, we provide a simple preface of fuzzy set theory. In Section 3, we propose a generalized decagonal fuzzy function and its $\sigma-$cut function. In section 4, we derive the ranking function. In Section 5, we show the fuzzy mathematical operations of generalized decagonal fuzzy numbers. In Section 6, we show the mathematical model of fully fuzzy linear programming problems. In section 7, we develop an algorithm of the fully fuzzy simplex method. In section 8, we give a numerical example. Finally, in Section 9, we conclude our paper.

2 Preface of Fuzzy Set theory

This section includes some basic definitions.

Definition 2.1. *[8] Let X be a nonempty set. A fuzzy set $\tilde{A}$ is characterized by its membership function:*
$M_{\tilde{A}}(x) : X \to [0,1]$, *where* $\tilde{A}(x) = \{(x, M_{\tilde{A}}(x)) \,| X \to [0,1]\}$.

Definition 2.2. *[5] A $\sigma-$cut is the crisp set $\tilde{A}_\sigma$ of all elements x that satisfy* $\tilde{A}_\sigma = \{x \in X | M_{\tilde{A}}(x) \geq \sigma, \sigma \in [0,1]\}$.

Definition 2.3. *[8, 9] When the following properties hold true for the membership function of a fuzzy set $\tilde{A}$ defined on the universal set of real numbers R, we say that $\tilde{A}$ is a fuzzy number.*
i. There exists at least one x_0 in R with $M_{\tilde{A}}(x) = 1$ ($if\, M_{\tilde{A}}(x) = \omega, 0 < \omega \leq 1$ and we say that $\tilde{A}$ is a generalized fuzzy number).
ii. $M_{\tilde{A}}(x)$ is piecewise continuous.
iii. $\tilde{A}$ is a normal fuzzy set.

3 A Generalized Decagonal Membership Function is proposed

In this section, we propose a membership function $M_{\widetilde{A_{Dec}}}(x)$ of a Decagonal fuzzy number $\widetilde{A_{Dec}} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha_{10}; k_1, k_2, \omega)$, whereas $\alpha_1 \leq \alpha_2 \leq \alpha_3 \leq \alpha_4 \leq \alpha_5 \leq \alpha_6 \leq \alpha_7 \leq \alpha_8 \leq \alpha_9 \leq \alpha_{10} \in R$, $k_1, k_2 \in [0,1]$ and

$0 < k_1 < k_2 < \omega \leq 1$ the graph function as shown in Figure 1:

$$M_{\widetilde{A_{Dec}}}(x) = \begin{cases} 0 & x < \alpha_1 \\ k_1\left(\frac{x-\alpha_1}{\alpha_2-\alpha_1}\right) & \alpha_1 \leq x < \alpha_2 \\ k_1 & \alpha_2 \leq x < \alpha_3 \\ k_1 + (k_2 - k1)\left(\frac{x-\alpha_3}{\alpha_4-\alpha_3}\right) & \alpha_3 \leq x < \alpha_4 \\ k_2 + (\omega - k2)\left(\frac{x-\alpha_4}{\alpha_5-\alpha_4}\right) & \alpha_4 \leq x < \alpha_5 \\ \omega & \alpha_5 \leq x < \alpha_6 \\ \omega + (k_2 - \omega)\left(\frac{x-\alpha_6}{\alpha_7-\alpha_6}\right) & \alpha_6 \leq x < \alpha_7 \\ k_2 + (k_1 - k2)\left(\frac{x-\alpha_7}{\alpha_8-\alpha_7}\right) & \alpha_7 \leq x < \alpha_8 \\ k_1 & \alpha_8 \leq x < \alpha_9 \\ k_1\left(1 - \left(\frac{x-\alpha_9}{\alpha_{10}-\alpha_9}\right)\right) & \alpha_9 \leq x < \alpha_{10} \\ 0 & x > \alpha_{10} \end{cases}$$

The (σ-cut) function.

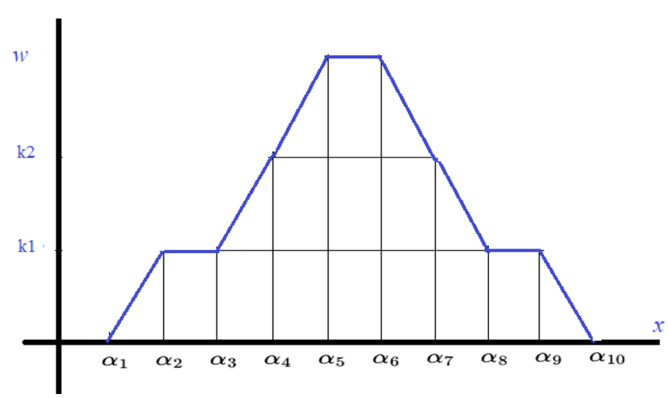

Figure 1: Generalized Decagonal Membership Function.

Let $\sigma \in [0, 1]$, the $(\sigma - cut)$ of decagonal fuzzy number

$\widetilde{A_{Dec}} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha_{10}; k_1, k_2, \omega)$ is defined as below:

$$\widetilde{A_{Dec\sigma}} = \begin{cases} \alpha_1 + \left(\frac{\sigma}{k_1}\right)(\alpha_2 - \alpha_1) & \sigma \in [0, k_1] \\ \alpha_3 + \left(\frac{\sigma - k_1}{k_2 - k_1}\right)(\alpha_4 - \alpha_3) & \sigma \in (k_1, k_2] \\ \alpha_4 + \left(\frac{\sigma - k_2}{\omega - k_2}\right)(\alpha_5 - \alpha_4) & \sigma \in (k_2, \omega] \\ \alpha_6 + \left(\frac{\sigma - \omega}{k_2 - \omega}\right)(\alpha_7 - \alpha_6) & \sigma \in (k_2, \omega] \\ \alpha_7 + \left(\frac{\sigma - k_2}{k_1 - k_2}\right)(\alpha_8 - \alpha_7) & \sigma \in (k_1, k_2] \\ \alpha_9 + \left(1 - \frac{\sigma}{k_1}\right)(\alpha_{10} - \alpha_9) & \sigma \in (0, k_1] \end{cases}$$

where at $\sigma \in [0, k_1]$
$(inf_1\widetilde{A_{Dec_\sigma}}, sup_3\widetilde{A_{Dec_\sigma}}) = (\left[\alpha_1 + \left(\frac{\sigma}{k_1}\right)(\alpha_2 - \alpha_1)\right], \left[\alpha_9 + \left(1 - \frac{\sigma}{k_1}\right)(\alpha_{10} - \alpha_9)\right])$
at $\sigma \in (k_1, k_2]$,
$(inf_2\widetilde{A_{Dec_\sigma}}, sup_2\widetilde{A_{Dec_\sigma}}) = (\left[\alpha_3 + \left(\frac{\sigma - k_1}{k_2 - k_1}\right)(\alpha_4 - \alpha_3)\right], \left[\alpha_7 + \left(\frac{\sigma - k_2}{k_1 - k_2}\right)(\alpha_8 - \alpha_7)\right])$
at $\sigma \in (k_2, \omega]$,
$(inf_3\widetilde{A_{Dec_\sigma}}, sup_1\widetilde{A_{Dec_\sigma}}) = (\left[\alpha_4 + \left(\frac{\sigma - k_2}{\omega - k_2}\right)(\alpha_5 - \alpha_4)\right], \left[\alpha_6 + \left(\frac{\sigma - \omega}{k_2 - \omega}\right)(\alpha_7 - \alpha_6\right]).$

4 Ranking Function

Fuzzy number ranking functions have significant practical value in statistical analysis. We are dealing with the collection of all fuzzy The collection function for fuzzy numbers defined over the real line is denoted by $F(R)$, and the map of every fuzzy number on the real line is denoted by $\Re : F(R) \to [0, 1]$ [10]. That can be used to convert every member of a fuzzy set into a real line. Here, we provide a ranking function that utilizes the linear decagonal membership function and a (σ-cut) function, where is an integer between zero and one. Let $\widetilde{A_{Dec}} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha_{10}; k_1, k_2, \omega)$, the proposed ranking function is:
$R\left(\widetilde{A_{Dec}}\right) = \int_0^\omega \left(\frac{3}{4}\right) inf_i\widetilde{A_{Dec_\sigma}} + \left(\frac{1}{4}\right) sup_j\widetilde{A_{Dec_\sigma}})d\sigma$
$i = 1, 2, 3 \quad j = \begin{cases} i+2 & i = 1 \\ i & i = 2 \\ i-2 & i = 3 \end{cases}$ Suppose that:

$$R\left(\widetilde{A_{Dec}}\right) = F_1 + F_2 + F_3, \tag{4.1}$$

where
$F_1 = \left(\frac{1}{3}\right) \int_0^{k_1} \left(\frac{3}{4}\right) \left[\alpha_1 + \left(\frac{\sigma}{k_1}\right)(\alpha_2 - \alpha_1)\right] d\sigma + \int_0^{k_1} \left(\frac{1}{4}\right) \left[\alpha_9 + \left(1 - \frac{\sigma}{k_1}\right)(\alpha_{10} - \alpha_9)\right] d\sigma$

$$F_1 = \frac{k_1}{24}(3\alpha_1 + 3\alpha_2 + \alpha_9 + \alpha_{10}) \quad (4.2)$$

$F_2 = \left(\frac{1}{3}\right) \int_{k_1}^{k_2} \left(\frac{3}{4}\right) \left[\alpha_3 + \left(\frac{\sigma - k_1}{k_2 - k_1}\right)(\alpha_4 - \alpha_3)\right] d\sigma + \int_{k_1}^{k_2} \left(\frac{1}{4}\right) \left[\alpha_7 + \left(\frac{\sigma - k_2}{k_1 - k_2}\right)(\alpha_8 - \alpha_7)\right] d\sigma$

$$F_2 = \frac{(k_2 - k_1)}{24}(3\alpha_3 + 3\alpha_4 + \alpha_7 + \alpha_8) \quad (4.3)$$

$F_3 = \left(\frac{1}{3}\right) \int_{k_2}^{\omega} \left(\frac{3}{4}\right) \left[\alpha_4 + \left(\frac{\sigma - k_2}{\omega - k_2}\right)(\alpha_5 - \alpha_4)\right] d\sigma + \int_{k_2}^{\omega} \left(\frac{1}{4}\right) \left[\alpha_6 + \left(\frac{\sigma - \omega}{k_2 - \omega}\right)(\alpha_7 - \alpha_6)\right] d\sigma$

$$F_3 = \frac{(\omega - k_2)}{24}(3\alpha_4 + 3\alpha_5 + \alpha_6 + \alpha_7) \quad (4.4)$$

Now substitute equations (4.2),(4.3),(4.4) into equation(4.1):
$\therefore \Re\left(\widetilde{A_{Dec}}\right) = \frac{1}{24}[3k_1\alpha_1 + 3k_1\alpha_2 + 3(k_2 - k_1)\alpha_3 + 3\left((k_2 - k_1) + (\omega - k_2)\right)\alpha_4 + 3(\omega - k_2)\alpha_5 + (\omega - k_2)\alpha_6 + \left((k_2 - k_1) + (\omega - k_2)\right)\alpha_7 + (k_2 - k_1)\alpha_8 + k_1\alpha_9 + k_1\alpha_{10}]$
$0 < k_1 < k_2 < \omega$, $0 < \omega \leq 1$

5 Fuzzy Mathematical Operations of Generalized Decagonal Fuzzy Numbers [11, 12]

Let $\widetilde{A_{Dec}}$ and $\widetilde{B_{Dec}}$ be two arbitrary generalized Decagonal fuzzy numbers such that;
$\widetilde{A_{Dec}} = (\alpha_1, \alpha_2, \alpha_3, ..., \alpha_{10}; k_1, l_1, \omega_1)$, $\widetilde{B_{Dec}} = (b_1, b_2, b_3, ..., b_{10}; k_2, l_2, \omega_2)$
Define the Addition, Subtraction, and multiplication operations [12] as follows:
•$\widetilde{A_{Dec}} \oplus \widetilde{B_{Dec}} = (\alpha_1 + b_1, \alpha_2 + b_2, \alpha_3 + b_3, ..., \alpha_{10} + b_{10}; min(k_1, k_2), min(l_1, l_2), min(\omega_1, \omega_2))$
•$\widetilde{A_{Dec}} \ominus \widetilde{B_{Dec}} = (\alpha_1 - b_{10}, \alpha_2 - b_9, \alpha_3 - b_8, ..., \alpha_9 - b_2, \alpha_{10} - b_1; min(k_1, k_2), min(l_1, l_2), min(\omega_1, \omega_2))$
•$\widetilde{A_{Dec}} \otimes \widetilde{B_{Dec}} = (\alpha_1 * b_1, \alpha_2 * b_2, \alpha_3 * b_3, ..., \alpha_9 * b_9, \alpha_{10} * b_{10}; min(k_1, k_2), min(l_1, l_2), min(\omega_1, \omega_2))$
•$\lambda \otimes \widetilde{A_{Dec}} = (\lambda\alpha_1, \lambda\alpha_2, \lambda\alpha_3, ..., \lambda\alpha_9, \lambda\alpha_{10}; k_1, l_1, \omega_1)$ if $\lambda > 0$
$= (\lambda\alpha_{10}, \lambda\alpha_9, \lambda\alpha_8, ..., \lambda\alpha_3, \lambda\alpha_2, \lambda\alpha_1; k_1, l_1, \omega_1)$ if $\lambda < 0$
• $if\ 0 \notin \left(\begin{smallmatrix} \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5 \\ \alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha_{10} \end{smallmatrix}\right)$ then;
$\frac{\left(\begin{smallmatrix} \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5 \\ \alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha_{10} \end{smallmatrix}\right)}{\left(\begin{smallmatrix} b_1, b_2, b_3, b_4, b_5 \\ b_6, b_7, b_8, b_9, b_{10} \end{smallmatrix}\right)} = \left(\frac{\alpha_1}{b_{10}}, \frac{\alpha_2}{b_9}, \frac{\alpha_3}{b_8}, ..., \frac{\alpha_{10}}{b_1}; min(k_1, k_2), min(l_1, l_2), min(\omega_1, \omega_2)\right)$

6 Fully Fuzzy Linear Programming Problem [2]

Consider the following FFLP problem: $\left(\substack{Max\\or\\min}\right)\widetilde{\omega} \cong \widetilde{C^T} \otimes \widetilde{X}$ such that
$\widetilde{A} \otimes \widetilde{X} \leq = \geq \widetilde{B}, \quad \widetilde{X} \geq \widetilde{0}$,
where $\widetilde{C^T} = \left(\widetilde{C}_j\right)_{1*n}, \widetilde{X} = (\widetilde{x}_j)_{1*n}, \widetilde{A} = (\widetilde{a}_{ij})_{m*n}, \widetilde{B} = \left(\widetilde{b}_i\right)_{m*1}$ and
$\widetilde{C^T}, \widetilde{X}, \widetilde{A}, \widetilde{B}$ are decagonal fuzzy numbers $\forall\ 1 \leq j \leq n,\ 1 \leq i \leq m$.

7 Develop an Algorithm for Fully Fuzzy Simplex Method

This section describes the solution steps of the FFLP problem.
• Convert all the inequalities of the constraints into equations by adding fuzzy slack variables and fuzzy surplus variables $\widetilde{S}_i, i = 1, 2, ..., m$.
• Construct the fully fuzzy simplex tableau in the following format Table 1:

Table 1: Fully fuzzy simplex tableau.

B.V	$\widetilde{x}_1$	$\widetilde{x}_2$	$\cdots$	$\widetilde{x}_n$	$\widetilde{S}_1$	$\widetilde{S}_2$	$\cdots$	$\widetilde{S}_n$	R.H.S	$\Re(R.H.S)$
$(\widetilde{\omega} \ominus \widetilde{c}_j)$	$-\widetilde{c}_1$	$-\widetilde{c}_2$	$\cdots$	$-\widetilde{c}_n$	(0,0,...,0)	(0,0,...,0)	$\cdots$	(0,0,...,0)	$\widetilde{\beta}_n$	$\Re(\widetilde{\beta}_n)$
$\widetilde{S}_1$	$\widetilde{a}_{11}$	$\widetilde{a}_{12}$	$\cdots$	$\widetilde{a}_{1n}$	(1,0,...,0)	(0,0,...,0)	$\cdots$	(0,0,...,0)	$\widetilde{b}_1$	$\Re(\widetilde{b}_1)$
$\widetilde{S}_2$	$\widetilde{a}_{21}$	$\widetilde{a}_{22}$	$\cdots$	$\widetilde{a}_{2n}$	(0,0,...,0)	(0,1,...,0)	$\cdots$	(0,0,...,0)	$\widetilde{b}_2$	$\Re(\widetilde{b}_2)$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$
$\widetilde{S}_m$	$\widetilde{a}_{m1}$	$\widetilde{a}_{m2}$	$\cdots$	$\widetilde{a}_{mn}$	(0,0,...,0)	(0,0,...,0)	$\cdots$	(0,0,...,1)	$\widetilde{b}_m$	$\Re(\widetilde{b}_m)$

• Use the ranking function technique to determine the entering variable (Select the most negative of $\Re(\widetilde{\omega} \ominus \widetilde{c}_j)$ for a maximum problem and the most positive for a minimum problem).
• Determine the decagonal fuzzy number that leaving the basic solution by the ratio: $\Theta = min\left\{\frac{\Re(\widetilde{b}_i)}{\Re(the\ elements\ of\ column\ entering\ variable)}\right\}.\ i = 1, 2, ..., m$
• Use arithmetic operations to get the new fuzzy tableau (new iteration) of a fully fuzzy simplex table.
• Repeat the steps even to get the optimal fuzzy solution (in max $\Re(\widetilde{\omega} \ominus \widetilde{c}_j) \geq 0$). In min $\Re(\widetilde{\omega} \ominus \widetilde{c}_j) \leq 0$.

8 Numerical Results

Consider the following fully fuzzy linear programming problem with generalized decagonal fuzzy number.
$Max\widetilde{\omega} = (1,2,3,4,5,6,7,8,9,10;k_1,k_2,\omega) \otimes \widetilde{x}_1 \oplus (2,3,4,5,6,7,8,9,11,12;$
$k_1,k_2,\omega)$ s.t: $(0,1,2,3,4,5,6,7,8,9;k_1,k_2,\omega) \otimes \widetilde{x}_1 \oplus (-1,1,3,4,5,6,7,8,9,10;$
$k_1,k_2,\omega) \otimes \widetilde{x}_2 \leq (1,2,4,6,10,12,14,16,20,27;k_1,k_2,\omega)$
$(1,2,3,4,5,6,7,8,9,10;k_1,k_2,\omega) \otimes \widetilde{x}_1 \oplus (-1,0,2,3,4,5,6,7,8,9;k_1,k_2,\omega) \otimes$
$\widetilde{x}_2 \leq (2,4,6,8,10,11,12,13,16,28;k_1,k_2,\omega)$

$$\widetilde{x}_1, \widetilde{x}_2 \geq 0 \;\; and \;\; 0 < k_1 < k_2 < \omega, 0 < \omega \leq 1$$

First, convert the problem to the standard form by adding the fuzzy slack variables:
$Max\widetilde{\omega} = (1,2,3,4,5,6,7,8,9,10;k_1,k_2,\omega) \otimes \widetilde{x}_1 \oplus (2,3,4,5,6,7,8,9,11,12;$
$k_1,k_2,\omega) \oplus \widetilde{0} \otimes \widetilde{s}_1 \oplus \widetilde{0} \otimes \widetilde{s}_2$ s.t
$(0,1,2,3,4,5,6,7,8,9;k_1,k_2,\omega) \otimes \widetilde{x}_1 \oplus (-1,1,3,4,5,6,7,8,9,10;k_1,k_2,\omega) \otimes$
$\widetilde{x}_2 \oplus (1,0,0,...,0) \otimes \widetilde{s}_1 = (1,2,4,6,10,12,14,16,20,27;k_1,k_2,\omega)$.
$(1,2,3,4,5,6,7,8,9,10;k_1,k_2,\omega) \otimes \widetilde{x}_1 \oplus (-1,0,2,3,4,5,6,7,8,9;k_1,k_2,\omega) \otimes$
$\widetilde{x}_2 \oplus (0,1,0,...,0) \otimes \widetilde{s}_2 = (2,4,6,8,10,11,12,13,16,28;k_1,k_2,\omega)$.

$$\widetilde{x}_1, \widetilde{x}_2, \widetilde{s}_1, \widetilde{s}_2 \geq 0.$$

Construct the table of fully fuzzy simplex method as shown in Table 2.

Table 2: Primary table of the fully fuzzy simplex method.

B.V	$\widetilde{x}_1$	$\widetilde{x}_2 \Downarrow$	$\widetilde{s}_1$	$\widetilde{s}_2$	R.H.S
$\widetilde{\omega}_1 \ominus \widetilde{c}_j$	$\begin{pmatrix} -10,-9,-8,-7,-6 \\ -5,-4,-3,-2,-1 \end{pmatrix}$	$\begin{pmatrix} -12,-11,-9,-8,-7 \\ -6,-5,-4,-3,-2 \end{pmatrix}$	$\begin{pmatrix} 0,0,0,0,0 \\ 0,0,0,0,0 \end{pmatrix}$	$\begin{pmatrix} 0,0,0,0,0 \\ 0,0,0,0,0 \end{pmatrix}$	$\begin{pmatrix} 0,0,0,0,0 \\ 0,0,0,0,0 \end{pmatrix}$
$\widetilde{s}_1 \Rightarrow$	$\begin{pmatrix} 0,1,2,3,4 \\ 5,6,7,8,9 \end{pmatrix}$	$\begin{pmatrix} -1,1,3,4,5 \\ 6,7,8,9,10 \end{pmatrix}$	$\begin{pmatrix} 1,0,0,0,0 \\ 0,0,0,0,0 \end{pmatrix}$	$\begin{pmatrix} 0,0,0,0,0 \\ 0,0,0,0,0 \end{pmatrix}$	$\begin{pmatrix} 1,2,4,6,10 \\ 12,14,16,20,27 \end{pmatrix}$
$\widetilde{s}_2$	$\begin{pmatrix} 1,2,3,4,5 \\ 6,7,8,9,10 \end{pmatrix}$	$\begin{pmatrix} -1,0,2,3,4 \\ 5,6,7,8,9 \end{pmatrix}$	$\begin{pmatrix} 0,0,0,0,0 \\ 0,0,0,0,0 \end{pmatrix}$	$\begin{pmatrix} 0,1,0,0,0 \\ 0,0,0,0,0 \end{pmatrix}$	$\begin{pmatrix} 2,4,6,8,10 \\ 11,12,13,16,28 \end{pmatrix}$

Now, by ranking function technique, determine the entering and leaving variables as follows:
$\Re\left(\widetilde{A_{Dec}}\right) = \frac{1}{24}[3k_1\alpha_1 + 3k_1\alpha_2 + 3(k_2-k_1)\alpha_3 + 3\left((k_2-k_1)+(\omega-k_2)\right)\alpha_4 + 3(\omega-k_2)\alpha_5 + (\omega-k_2)\alpha_6 + \left((k_2-k_1)+(\omega-k_2)\right)\alpha_7 + (k_2-k_1)\alpha_8 + k_1\alpha_9 + k_1\alpha_{10}]$
Since $0 < k_1 < k_2 < \omega, 0 < \omega < 1$
let $\omega = 0.9, k_1 = \frac{\omega}{3} = 0.3, k_2 = \frac{2\omega}{3} = 0.6$
$\Re\left(\widetilde{A_{Dec}}\right) = \frac{1}{24}[0.9\alpha_1 + 0.9\alpha_2 + 0.9\alpha_3 + 1.8\alpha_4 + 0.9\alpha_5 + 0.3\alpha_6 + 0.6\alpha_7 + 0.3\alpha_8 + 0.3\alpha_9 + 0.3\alpha_{10}]$

$\Re\left(\widetilde{A_{Dec}}\right) = \frac{1}{8}[0.3(\alpha_1 + \alpha_2 + \alpha_3) + 0.6\alpha_4 + 0.3\alpha_5 + 0.1\alpha_6 + 0.2\alpha_7 + 0.1(\alpha_8 + \alpha_9 + \alpha_{10})]$

The entering variable $= min\left\{\Re(\widetilde{x}_1), \Re(\widetilde{x}_2)\right\} = min\left\{-2, -2.37\right\} = -2.37$

The leaving variable $= min\left\{\frac{\Re\binom{1,2,4,6,10}{12,14,16,20,27}}{\Re\binom{-1,1,3,4,5}{6,7,8,9,10}}, \frac{\Re\binom{2,4,6,8,10}{11,12,13,16,28}}{\Re\binom{-1,0,2,3,4}{5,6,7,8,9}}\right\} =$

$min\left\{\frac{2.37}{1.18}, \frac{2.57}{0.9}\right\} = \{2.008, 2.855\} = 2.008$

$\therefore$ The entering variable is $\widetilde{x}_2$ and the leaving variable is $\widetilde{s}_1$, The pivot element of decagonal fuzzy numbers is: $\binom{-1,1,3,4,5}{6,7,8,9,10}$. Using the operation of the decagonal fuzzy number to find a new iteration of the fully fuzzy simplex method, Therefore Table 3 is the new table of the problem:

Table 3: The optimal table solution for the fully fuzzy simplex method.

B.V	$\widetilde{x}_1$	$\widetilde{x}_2$	$\widetilde{s}_1$	$\widetilde{s}_2$	R.H.S
$\widetilde{\omega}_1 \ominus \widetilde{c}_j$	$\binom{-10,2,14,26,38}{50,62,74,86,98}$	$\binom{0,12,24,36,48}{60,72,84,96,108}$	$\binom{11,0,0,0,0}{0,0,0,0,0}$	$\binom{0,0,0,0,0}{0,0,0,0,0}$	$\binom{11,22,44,66,110}{132,154,176,220,297}$
$\widetilde{x}_2$	$\binom{0,1,2,3,4}{5,6,7,8,9}$	$\binom{-1,1,3,4,5}{6,7,8,9,10}$	$\binom{1,0,0,0,0}{0,0,0,0,0}$	$\binom{0,0,0,0,0}{0,0,0,0,0}$	$\binom{1,2,4,6,10}{12,14,16,20,27}$
$\widetilde{s}_2$	$\binom{1,2,3,4,5}{6,7,8,9,10}$	$\binom{-1,0,2,3,4}{5,6,7,8,9}$	$\binom{0,0,0,0,0}{0,0,0,0,0}$	$\binom{0,1,0,0,0}{0,0,0,0,0}$	$\binom{2,4,6,8,10}{11,12,13,16,28}$

Since $\Re(\widetilde{\omega}_1 \ominus \widetilde{c}_j) \geq 0$ and $\Re(R.H.S) \geq 0$ in Table 3, the optimal solution is reached: $\widetilde{x}_1 = \widetilde{0}, \widetilde{x}_2 = (1, 2, 4, 6, 10, 12, 14, 16, 20, 27; 0.3, 0.6, 0.9)$,
Max $\widetilde{\omega}_1 = (11, 22, 44, 66, 110, 132, 154, 176, 220, 297; 0.3, 0.6, 0.9)$.
Then, by using ranking function technique max $\widetilde{\omega} = \Re(\widetilde{\omega}_1)$.

9 Conclusion

The proposed generalized decagonal membership function with a new ranking function enables us to obtain the optimal fuzzy solution to FFLP problems, which is especially helpful due to the programming program's fuzziness and its applicability in cases where the coefficients are fuzzy statistics to help of real-world occurrences and cannot be determined with precision. For decagonal fuzzy numbers, the fuzzy number method of the fully fuzzy simplex technique was introduced, which may be used to determine which fuzzy solution is best for the FFLP problem. As width previous iterations of the fully fuzzy simplex technique, the ranking function was used to determine which variables should be left and which should be eliminated to arrive at the optimal solution.

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