International Journal of Mathematics and
Computer Science, **18**(2023), no. 4, 755–759

On $\delta p(\Lambda, s)$-symmetric spaces

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(Submitted January 25, 2023, Accepted May 22, 2023,
Published May 31, 2023)

Abstract

In this paper, we deal with the notion of $\delta p(\Lambda, s)$-symmetric spaces. Moreover, some characterizations $\delta p(\Lambda, s)$-symmetric spaces are investigated.

1 Introduction

Mashhour et al. [7] introduced and studied the concept of preopen sets which is weaker than the notion of open sets in topological spaces. Veličko [10] introduced δ-open sets, which are stronger than open sets. Raychaudhuri and Mukherjee [8] introduced and investigated the notions of δ-preopen sets and δ-almost continuity in topological spaces. The class of δ-preopen sets is larger than that of preopen sets. Caldas et al. [3] introduced some weak separation axioms by utilizing the notions of δ-preopen sets and the δ-preclosure operator. The concept of semi-open sets was first introduced by Levine [6]. Caldas and Dontchev [5] introduced and investigated the concepts of Λ_s-sets and V_s-sets in topological spaces. Moreover, Caldas et al. [4] studied the notion of δ-Λ_s-semiclosed sets which is defined as the intersection of a δ-Λ_s-set and a

Key words and phrases: $\delta p(\Lambda, s)$-open set, $\delta p(\Lambda, s)$-symmetric space.
Corresponding author: Jeeranunt Khampakdee.
AMS (MOS) Subject Classifications: 54A05, 54D10.
ISSN 1814-0432, 2023, http://ijmcs.future-in-tech.net

δ-semiclosed set. Boonpok and Viriyapong [2] introduced and investigated the concept of (Λ, s)-closed sets by utilizing the notions of Λ_s-sets and semi-closed sets. In this paper, we introduced the concept of $\delta p(\Lambda, s)$-symmetric spaces. Furthermore, several characterizations of $\delta p(\Lambda, s)$-symmetric spaces are discussed.

2 Preliminaries

Throughout the paper, unless explicitly stated, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms. Let A be a subset of a topological space (X, τ). The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A of a topological space (X, τ) is called *semi-open* [6] if $A \subseteq \mathrm{Cl}(\mathrm{Int}(A))$. The complement of a semi-open set is called *semi-closed*. The family of all semi-open sets in a topological space (X, τ) is denoted by $SO(X, \tau)$. A subset A^{Λ_s} [5] (resp. A^{Λ_s}) is defined as follows: $A^{\Lambda_s} = \cap\{U \mid U \supseteq A, U \in SO(X, \tau)\}$. A subset A of a topological space (X, τ) is called a Λ_s-*set* if $A = A^{\Lambda_s}$. A subset A of a topological space (X, τ) is called (Λ, s)-*closed* [2] if $A = T \cap C$, where T is a Λ_s-set and C is a semi-closed set. The complement of a (Λ, s)-closed set is called (Λ, s)-*open*. The family of all (Λ, s)-closed (resp. (Λ, s)-open) sets in a topological space (X, τ) is denoted by $\Lambda_s C(X, \tau)$ (resp. $\Lambda_s O(X, \tau)$). Let A be a subsets of a topological space (X, τ). A point $x \in X$ is called a (Λ, s)-*cluster point* [2] of A if for every (Λ, s)-open set U of X containing x we have $A \cap U \neq \emptyset$. The set of all (Λ, s)-cluster points of A is called the (Λ, s)-*closure* [2] of A and is denoted by $A^{(\Lambda,s)}$. The union of all (Λ, s)-open sets contained in A is called the (Λ, s)-*interior* [2] of A and is denoted by $A_{(\Lambda,s)}$. Let A be a subset of a topological space (X, τ). A point x of X is called a $\delta(\Lambda, s)$-*cluster point* [9] of A if $A \cap [V^{(\Lambda,s)}]_{(\Lambda,s)} \neq \emptyset$ for every (Λ, s)-open set V of X containing x. The set of all $\delta(\Lambda, s)$-cluster points of A is called the $\delta(\Lambda, s)$-*closure* [9] of A and is denoted by $A^{\delta(\Lambda,s)}$. If $A = A^{\delta(\Lambda,s)}$, then A is said to be $\delta(\Lambda, s)$-*closed* [9]. The complement of a $\delta(\Lambda, s)$-closed set is said to be $\delta(\Lambda, s)$-*open* [9]. The union of all $\delta(\Lambda, s)$-open sets contained in A is called the $\delta(\Lambda, s)$-*interior* [9] of A and is denoted by $A_{\delta(\Lambda,s)}$. A subset A of a topological space (X, τ) is said to be $\delta p(\Lambda, s)$-*open* [9] if $A \subseteq [A^{(\Lambda,s)}]_{\delta(\Lambda,s)}$. The complement of a $\delta p(\Lambda, s)$-open set is said to be $\delta p(\Lambda, s)$-*closed* [9]. The family of all $\delta p(\Lambda, s)$-open (resp. $\delta p(\Lambda, s)$-closed) sets in a topological space (X, τ) is denoted by $\delta p(\Lambda, s)O(X, \tau)$ (resp. $\delta p(\Lambda, s)C(X, \tau)$). Let A be a subset of a topological space (X, τ). The intersection of all $\delta p(\Lambda, s)$-closed sets containing A is called $\delta p(\Lambda, s)$-*closure* [9] of A and is denoted by $A^{\delta p(\Lambda,s)}$.

Lemma 2.1. [9] *For the $\delta p(\Lambda, s)$-closure of subsets A, B in a topological space (X, τ), the following properties hold:*

(1) If $A \subseteq B$, then $A^{\delta p(\Lambda,s)} \subseteq B^{\delta p(\Lambda,s)}$.

(2) A is $\delta p(\Lambda, s)$-closed in (X, τ) if and only if $A = A^{\delta p(\Lambda,s)}$.

(3) $A^{\delta p(\Lambda,s)}$ is $\delta p(\Lambda, s)$-closed, that is, $A^{\delta p(\Lambda,s)} = [A^{\delta p(\Lambda,s)}]^{\delta p(\Lambda,s)}$.

(4) $x \in A^{\delta p(\Lambda,s)}$ if and only if $A \cap V \neq \emptyset$ for every $V \in \delta p(\Lambda, s)O(X, \tau)$ containing x.

3 $\delta p(\Lambda, s)$-symmetric spaces

In this section, we introduce the concept of $\delta p(\Lambda, s)$-symmetric spaces. Moreover, some characterizations of $\delta p(\Lambda, s)$-symmetric spaces are discussed.

Definition 3.1. *A topological space (X, τ) is said to be $\delta p(\Lambda, s)$-T_1 if, for any distinct pair of points x and y of X, there exist a $\delta p(\Lambda, s)$-open set U of X containing x but not y and a $\delta p(\Lambda, s)$-open set V of X containing y but not x.*

Theorem 3.2. *A topological space (X, τ) is $\delta p(\Lambda, s)$-T_1 if and only if the singletons are $\delta p(\Lambda, s)$-closed sets.*

Proof. Suppose that (X, τ) is $\delta p(\Lambda, s)$-T_1 and x be any point of X. Let $y \in X - \{x\}$. Then, $x \neq y$ and so there exists a $\delta p(\Lambda, s)$-open set U_y such that $y \in U_y$ but $x \notin U_y$. Thus, $y \in U_y \subseteq X - \{x\}$ and hence $X - \{x\} = \cup\{U_y \mid y \in (X - \{x\})\}$ is $\delta p(\Lambda, s)$-open.

Conversely, suppose that $\{z\}$ is $\delta p(\Lambda, s)$-closed for each $z \in X$. Let $x, y \in X$ such that $x \neq y$. Now $x \neq y$ implies $y \in X - \{x\}$. Thus, $X - \{x\}$ is a $\delta p(\Lambda, s)$-open set containing y but not containing x. Similarly, $X - \{y\}$ is a $\delta p(\Lambda, s)$-open set containing x but not containing y. This shows that (X, τ) is $\delta p(\Lambda, s)$-T_1. □

Definition 3.3. *A topological space (X, τ) is called $\delta p(\Lambda, s)$-symmetric if, for each x and y in X, $x \in \{y\}^{\delta p(\Lambda,s)}$ implies $y \in \{y\}^{\delta p(\Lambda,s)}$.*

Lemma 3.4. *Let (X, τ) be a topological space. For each point $x \in X$, $\{x\}$ is $p(\Lambda, s)$-open or $p(\Lambda, s)$-closed.*

Theorem 3.5. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) *is* $\delta p(\Lambda, s)$*-symmetric.*

(2) For each $x \in X$, $\{x\}$ *is* $\delta p(\Lambda, s)$*-closed.*

(3) (X, τ) *is* $\delta p(\Lambda, s)$-T_1.

Proof. (1) ⇒ (2): Suppose that (X, τ) is $\delta p(\Lambda, s)$-symmetric. Let x be any point of X and y be any distinct point from x. By Lemma 3.4, $\{y\}$ is $p(\Lambda, s)$-open or $p(\Lambda, s)$-closed in (X, τ). (i) In case $\{y\}$ is $p(\Lambda, s)$-open, put $V_y = \{y\}$, then $V_y \in \delta p(\Lambda, s)O(X, \tau)$. (ii) In case $\{y\}$ is $p(\Lambda, s)$-closed, $x \notin \{y\} = \{y\}^{p(\Lambda,s)}$ and $x \notin \{y\}^{\delta p(\Lambda,s)}$. By (1), $y \notin \{x\}^{\delta p(\Lambda,s)}$. Now put $V_y = X - \{x\}^{\delta p(\Lambda,s)}$. Then, $x \notin V_y$, $y \in V_y$ and $V_y \in \delta p(\Lambda, s)O(X, \tau)$. Thus, $X - \{x\} = \underset{y \in X - \{x\}}{\cup} V_y \in \delta p(\Lambda, s)O(X, \tau)$ and hence $\{x\}$ is $\delta p(\Lambda, s)$-closed.

(2) ⇒ (3): Suppose that $\{z\}$ is $\delta p(\Lambda, s)$-closed for each $z \in X$. Let $x, y \in X$ with $x \neq y$. Now $x \neq y$ implies $y \in X - \{x\}$. Thus, $X - \{x\}$ is a $\delta p(\Lambda, s)$-open set containing y but not containing x. Similarly, we have $X - \{y\}$ is a $\delta p(\Lambda, s)$-open set containing x but not containing y. This shows that (X, τ) is $\delta p(\Lambda, s)$-T_1.

(3) ⇒ (1): Suppose that $y \notin \{x\}^{\delta p(\Lambda,s)}$. Then, since $x \neq y$, by (3) there exists a $\delta p(\Lambda, s)$-open set U containing x such that $y \notin U$ and hence $x \notin \{y\}^{\delta p(\Lambda,s)}$. This shows that $x \in \{y\}^{\delta p(\Lambda,s)}$ implies $y \in \{x\}^{\delta p(\Lambda,s)}$. Thus, (X, τ) is $\delta p(\Lambda, s)$-symmetric. □

Definition 3.6. [1] *A subset A of a topological space (X, τ) is said to be generalized $\delta p(\Lambda, s)$-closed (briefly, g-$\delta p(\Lambda, s)$-closed) if $A^{\delta p(\Lambda,s)} \subseteq U$ whenever $A \subseteq U$ and U is $\delta p(\Lambda, s)$-open in (X, τ).*

Theorem 3.7. *A topological space (X, τ) is $\delta p(\Lambda, s)$-symmetric if and only if $\{x\}$ is g-$\delta p(\Lambda, s)$-closed for each $x \in X$.*

Proof. Assume that $x \in \{y\}^{\delta p(\Lambda,s)}$ but $y \in \{x\}^{\delta p(\Lambda,s)}$. This means that the complement of $\{x\}^{\delta p(\Lambda,s)}$ contains y. Thus, the set $\{y\}$ is a subset of the complement of $\{x\}^{\delta p(\Lambda,s)}$. This implies that $\{y\}^{\delta p(\Lambda,s)}$ is a subset of the complement of $\{x\}^{\delta p(\Lambda,s)}$. Now the complement of $\{x\}^{\delta p(\Lambda,s)}$ contains x which is a contradiction.

Conversely, suppose that $\{x\} \subseteq U \in \delta p(\Lambda, s)O(X, \tau)$, but $\{x\}^{\delta p(\Lambda,s)}$ is not a subset of U. This means that $\{x\}^{\delta p(\Lambda,s)}$ and the complement of U are not disjoint. Let y belongs to their intersection. Now we have $x \in \{y\}^{\delta p(\Lambda,s)}$ which is a subset of the complement of U and $x \notin U$. This is a contradiction. □

Acknowledgment. This research project was financially supported by Mahasarakham University.

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