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On characterizations of $\delta p(\Lambda, s)$-$\mathscr{D}_1$ spaces

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Abstract

Our main purpose is to introduce the concept of $\delta p(\Lambda, s)$-$\mathscr{D}_1$ spaces. Moreover, some characterizations of $\delta p(\Lambda, s)$-$\mathscr{D}_1$ spaces are investigated.

1 Introduction

In 1968, Veličko [8] introduced δ-open sets, which are stronger than open sets. In 1982, Mashhour et al. [6] introduced and investigated the notion of preopen sets which is weaker than the notion of open sets in topological spaces. In 1993, Raychaudhuri and Mukherjee [7] introduced and studied the notions of δ-preopen sets and δ-almost continuity in topological spaces. The class of δ-preopen sets is larger than that of preopen sets. In 2005, Caldas et al. [2] introduced some weak separation axioms by utilizing the notions of δ-preopen sets and the δ-preclosure operator. The concept of semi-open sets was first introduced by Levine [5]. Caldas and Dontchev [4] introduced and investigated the notion of Λ_s-sets in topological spaces. Moreover, Caldas et al. [3] studied further the notion of δ-Λ_s-semiclosed sets which is defined as the intersection of a δ-Λ_s-set and a δ-semiclosed set. In [1], the present

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authors introduced and studied the notion of (Λ, s)-closed sets by utilizing the notions of Λ_s-sets and semi-closed sets. In this paper, we introduced the concept of $\delta p(\Lambda, s)$-$\mathscr{D}_1$ spaces. Moreover, several characterizations of $\delta p(\Lambda, s)$-$\mathscr{D}_1$ spaces are discussed.

2 Preliminaries

Throughout the present paper, unless explicitly stated, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assume . Let A be a subset of a topological space (X, τ). The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A of a topological space (X, τ) is called *semi-open* [5] if $A \subseteq \mathrm{Cl}(\mathrm{Int}(A))$. The complement of a semi-open set is called *semi-closed*. The family of all semi-open (resp. semi-closed) sets in a topological space (X, τ) is denoted by $SO(X, \tau)$ (resp. $SC(X, \tau)$). A subset A^{Λ_s} (resp. A^{V_s}) [2] is defined as follows: $A^{\Lambda_s} = \cap\{U \mid U \supseteq A, U \in SO(X, \tau)\}$ (resp. $A^{V_s} = \cup\{F \mid F \subseteq A, F \in SC(X, \tau)\}$). A subset A of a topological space (X, τ) is called a *Λ_s-set* (resp. *V_s-set*) [2] if $A = A^{\Lambda_s}$ (resp. $A = A^{V_s}$). A subset A of a topological space (X, τ) is called *(Λ, s)-closed* [1] if $A = T \cap C$, where T is a Λ_s-set and C is a semi-closed set. The complement of a (Λ, s)-closed set is called *(Λ, s)-open*. The family of all (Λ, s)-closed (resp. (Λ, s)-open) sets in a topological space (X, τ) is denoted by $\Lambda_s C(X, \tau)$ (resp. $\Lambda_s O(X, \tau)$). Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a *(Λ, s)-cluster point* [1] of A if for every (Λ, s)-open set U of X containing x we have $A \cap U \neq \emptyset$. The set of all (Λ, s)-cluster points of A is called the *(Λ, s)-closure* [1] of A and is denoted by $A^{(\Lambda, s)}$. The union of all (Λ, s)-open sets contained in A is called the *(Λ, s)-interior* [1] of A and is denoted by $A_{(\Lambda, s)}$.

3 $\delta p(\Lambda, s)$-$\mathscr{D}_1$ spaces

We introduce the notion of $\delta p(\Lambda, s)$-$\mathscr{D}_1$ spaces. Moreover, some characterizations of $\delta p(\Lambda, s)$-$\mathscr{D}_1$ spaces are discussed.

Definition 3.1. *Let A be a subset of a topological space (X, τ). A point x of X is called a $\delta(\Lambda, s)$-cluster point of A if $A \cap [V^{(\Lambda, s)}]_{(\Lambda, s)} \neq \emptyset$ for every (Λ, s)-open set V of X containing x. The set of all $\delta(\Lambda, s)$-cluster points of A is called the $\delta(\Lambda, s)$-closure of A and is denoted by $A^{\delta(\Lambda, s)}$. If $A = A^{\delta(\Lambda, s)}$,* *then A is said to be $\delta(\Lambda, s)$-closed. The complement of a $\delta(\Lambda, s)$-closed set is said to be $\delta(\Lambda, s)$-open.*

Let A be a subset of a topological space (X, τ). The union of all $\delta(\Lambda, s)$-open sets contained in A is called the $\delta(\Lambda, s)$-*interior* of A and is denoted by $A_{\delta(\Lambda,s)}$.

Definition 3.2. *A subset A of a topological space (X, τ) is said to be $\delta p(\Lambda, s)$-open if $A \subseteq [A^{(\Lambda,s)}]_{\delta(\Lambda,s)}$. The complement of a $\delta p(\Lambda, s)$-open set is said to be $\delta p(\Lambda, s)$-closed.*

The family of all $\delta p(\Lambda, s)$-open (resp. $\delta p(\Lambda, s)$-closed) sets in a topological space (X, τ) is denoted by $\delta p(\Lambda, s)O(X, \tau)$ (resp. $\delta p(\Lambda, s)C(X, \tau)$). Let A be a subset of a topological space (X, τ). The intersection of all $\delta p(\Lambda, s)$-closed sets containing A is called the $\delta p(\Lambda, s)$-*closure* of A and is denoted by $A^{\delta p(\Lambda,s)}$.

Lemma 3.3. *For the $\delta p(\Lambda, s)$-closure of subsets A, B in a topological space (X, τ), the following properties hold:*

(1) If $A \subseteq B$, then $A^{\delta p(\Lambda,s)} \subseteq B^{\delta p(\Lambda,s)}$.

(2) A is $\delta p(\Lambda, s)$-closed in (X, τ) if and only if $A = A^{\delta p(\Lambda,s)}$.

(3) $A^{\delta p(\Lambda,s)}$ is $\delta p(\Lambda, s)$-closed, that is, $A^{\delta p(\Lambda,s)} = [A^{\delta p(\Lambda,s)}]^{\delta p(\Lambda,s)}$.

(4) $x \in A^{\delta p(\Lambda,s)}$ if and only if $A \cap V \neq \emptyset$ for every $V \in \delta p(\Lambda, s)O(X, \tau)$ containing x.

Lemma 3.4. *For a family $\{A_\gamma \mid \gamma \in \nabla\}$ of a topological space (X, τ), the following properties hold:*

(1) $[\cap\{A_\gamma \mid \gamma \in \nabla\}]^{\delta p(\Lambda,s)} \subseteq \cap\{A_\gamma^{\delta p(\Lambda,s)} \mid \gamma \in \nabla\}$.

(2) $[\cup\{A_\gamma \mid \gamma \in \nabla\}]^{\delta p(\Lambda,s)} \supseteq \cup\{A_\gamma^{\delta p(\Lambda,s)} \mid \gamma \in \nabla\}$.

Definition 3.5. *A subset N_x of a topological space (X, τ) is said to be a $\delta p(\Lambda, s)$-neighborhood of a point $x \in X$ if there exists a $\delta p(\Lambda, s)$-open set U such that $x \in U \subseteq N_x$.*

Definition 3.6. *Let (X, τ) be a topological space. A point $x \in X$ which has only X as the $\delta p(\Lambda, s)$-neighborhood is called a $\delta p(\Lambda, s)$-neat point.*

Lemma 3.7. *Let (X, τ) be a topological space. For each point $x \in X$, $\{x\}$ is $p(\Lambda, s)$-open or $p(\Lambda, s)$-closed.*

Lemma 3.8. *Let A be a subset of a topological space (X, τ). If A is $p(\Lambda, s)$-open in (X, τ), then A is $\delta p(\Lambda, s)$-open in (X, τ).*

Definition 3.9. *A subset A of a topological space (X, τ) is called a $\delta p(\Lambda, s)\mathscr{D}$-set if there exist $\delta p(\Lambda, s)$-open sets U and V such that $U \neq X$ and $A = U - V$.*

Definition 3.10. *A topological space (X, τ) is said to be $\delta p(\Lambda, s)$-$\mathscr{D}_1$ if, for any distinct pair of points x and y of X, there exist a $\delta p(\Lambda, s)\mathscr{D}$-set U of X containing x but not y and a $\delta p(\Lambda, s)\mathscr{D}$-set V of X containing y but not x.*

Theorem 3.11. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is $\delta p(\Lambda, s)$-$\mathscr{D}_1$;

(2) (X, τ) has no $\delta p(\Lambda, s)$-neat point.

Proof. (1) $\Rightarrow$ (2): Since (X, τ) is $\delta p(\Lambda, s)$-$\mathscr{D}_1$, each point x of X is contained in a $\delta p(\Lambda, s)\mathscr{D}$-set $G = U - V$ and thus in U, where U and V are $\delta p(\Lambda, s)$-open sets. By definition $U \neq X$. This implies that x is not a $\delta p(\Lambda, s)$-neat point.

(2) $\Rightarrow$ (1): By Lemma 3.7, for each distinct pair of points $x, y \in X$, at least one of them, x (say) has a $\delta p(\Lambda, s)$-neighborhood U containing x and not y. Thus U which is different from X is a $\delta p(\Lambda, s)\mathscr{D}$-set. If X has no $\delta p(\Lambda, s)$-neat point, then y is not a $\delta p(\Lambda, s)$-neat point. This means that there exists a $\delta p(\Lambda, s)$-neighborhood V of y such that $V \neq X$. Thus $y \in V - U$ but not y and $V - U$ is a $\delta p(\Lambda, s)\mathscr{D}$-set. This shows that (X, τ) is $\delta p(\Lambda, s)$-$\mathscr{D}_1$. □

Definition 3.12. *A function $f : (X, \tau) \to (Y, \sigma)$ is called $\delta p(\Lambda, s)$-continuous if, for each $x \in X$ and each $\delta p(\Lambda, s)$-open set V of Y containing $f(x)$, there exists a $\delta p(\Lambda, s)$-open set U of X containing x such that $f(U) \subseteq V$.*

Lemma 3.13. *A function $f : (X, \tau) \to (Y, \sigma)$ is $\delta p(\Lambda, s)$-continuous if and only if $f^{-1}(V)$ is $\delta p(\Lambda, s)$-open in X for every $\delta p(\Lambda, s)$-open set V of Y.*

Theorem 3.14. *If $f : (X, \tau) \to (Y, \sigma)$ is a $\delta p(\Lambda, s)$-continuous surjective function and B is a $\delta p(\Lambda, s)\mathscr{D}$-set in Y, then $f^{-1}(B)$ is a $\delta p(\Lambda, s)\mathscr{D}$-set in X.*

Proof. Let B be a $\delta p(\Lambda, s)\mathscr{D}$-set in Y. Then, there exist $\delta p(\Lambda, s)$-open sets U and V in Y such that $B = U - V$ and $U \neq Y$. By the $\delta p(\Lambda, s)$-continuity of f, $f^{-1}(U)$ and $f^{-1}(V)$ are $\delta p(\Lambda, s)$-open in X. Since $U \neq Y$, we have $f^{-1}(U) \neq X$. Thus $f^{-1}(B) = f^{-1}(U) - f^{-1}(V)$ is a $\delta p(\Lambda, s)\mathscr{D}$-set. □

Theorem 3.15. *If (Y, σ) is a $\delta p(\Lambda, s)$-$\mathscr{D}_1$ space and $f : (X, \tau) \to (Y, \sigma)$ is a $\delta p(\Lambda, s)$-continuous bijection, then (X, τ) is $\delta p(\Lambda, s)$-$\mathscr{D}_1$.*

Proof. Suppose that (Y, σ) is a $\delta p(\Lambda, s)$-$\mathscr{D}_1$ space. Let x and y be any pair of distinct points in X. Since f is injective and (Y, σ) is $\delta p(\Lambda, s)$-$\mathscr{D}_1$, there exist $\delta p(\Lambda, s)\mathscr{D}$-sets U and V of Y containing $f(x)$ and $f(y)$, respectively, such that $f(y) \notin U$ and $f(x) \notin V$. By Theorem 3.14, $f^{-1}(U)$ and $f^{-1}(V)$ are $\delta p(\Lambda, s)\mathscr{D}$-sets in X containing x and y, respectively, such that $y \notin f^{-1}(U)$ and $x \notin f^{-1}(V)$. This shows that (X, τ) is $\delta p(\Lambda, s)$-$\mathscr{D}_1$. □

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