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Sober $\delta p(\Lambda, s)$-R_0 spaces

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Abstract

This paper is concerned with the concept of sober $\delta p(\Lambda, s)$-R_0 spaces. Moreover, several characterizations of sober $\delta p(\Lambda, s)$-R_0 spaces are investigated.

1 Introduction

In 1963, Levine [6] offered a new concept in the field of topology by introducing the notion of semi-open sets in topological spaces. In 1968, Veličko [11] introduced δ-open sets, which are stronger than open sets. In 1982, Mashhour et al. [7] introduced and studied the concept of preopen sets. In 1993, Raychaudhuri and Mukherjee [9] introduced the notions of δ-preopen sets and δ-preclosures. Moreover, Caldas et al. [2] introduced some weak separation axioms by utilizing the notions of δ-preopen sets and the δ-preclosure operator. In 1997, Park et al. [8] introduced δ-semiopen sets which are stronger than semi-open sets but weaker than δ-open sets. In 1998, Caldas and Dontchev [5] introduced and investigated the notions of Λ_s-sets and V_s-sets in topological spaces. In 2003, Caldas et al. [4] investigated some weak

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separation axioms by utilizing δ-semiopen sets and the δ-semiclosure operator. In 2005, Caldas et al. [3] investigated the notion of δ-Λ_s-semiclosed sets which is defined as the intersection of a δ-Λ_s-set and a δ-semiclosed set. In [1], the present authors introduced and investigated the concept of (Λ, s)-closed sets by utilizing the notions of Λ_s-sets and semi-closed sets. In this paper, we introduce the concept of sober $\delta p(\Lambda, s)$-R_0 spaces. Moreover, we characterize sober $\delta p(\Lambda, s)$-R_0 spaces.

2 Preliminaries

Throughout this paper, unless explicitly stated, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed.

Let A be a subset of a topological space (X, τ). The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A of a topological space (X, τ) is called *semi-open* [6] if $A \subseteq \mathrm{Cl}(\mathrm{Int}(A))$. The complement of a semi-open set is called *semi-closed*. The family of all semi-open (resp. semi-closed) sets in a topological space (X, τ) is denoted by $SO(X, \tau)$ (resp. $SC(X, \tau)$). A subset A^{Λ_s} (resp. A^{V_s}) [5] is defined as follows: $A^{\Lambda_s} = \cap\{U \mid U \supseteq A, U \in SO(X, \tau)\}$ (resp. $A^{V_s} = \cup\{F \mid F \subseteq A, F \in SC(X, \tau)\}$). A subset A of a topological space (X, τ) is called a Λ_s*-set* (resp. V_s*-set*) [5] if $A = A^{\Lambda_s}$ (resp. $A = A^{V_s}$). A subset A of a topological space (X, τ) is called (Λ, s)*-closed* [1] if $A = T \cap C$, where T is a Λ_s-set and C is a semi-closed set. The complement of a (Λ, s)-closed set is called (Λ, s)*-open*. The family of all (Λ, s)-closed (resp. (Λ, s)-open) sets in a topological space (X, τ) is denoted by $\Lambda_s C(X, \tau)$ (resp. $\Lambda_s O(X, \tau)$). Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a (Λ, s)*-cluster point* [1] of A if for every (Λ, s)-open set U of X containing x we have $A \cap U \neq \emptyset$. The set of all (Λ, s)-cluster points of A is called the (Λ, s)*-closure* [1] of A and is denoted by $A^{(\Lambda,s)}$. The union of all (Λ, s)-open sets contained in A is called the (Λ, s)*-interior* [1] of A and is denoted by $A_{(\Lambda,s)}$. Let A be a subset of a topological space (X, τ). A point x of X is called a $\delta(\Lambda, s)$*-cluster point* [10] of A if $A \cap [V^{(\Lambda,s)}]_{(\Lambda,s)} \neq \emptyset$ for every (Λ, s)-open set V of X containing x. The set of all $\delta(\Lambda, s)$-cluster points of A is called the $\delta(\Lambda, s)$*-closure* [10] of A and is denoted by $A^{\delta(\Lambda,s)}$. If $A = A^{\delta(\Lambda,s)}$, then A is said to be $\delta(\Lambda, s)$*-closed* [10]. The complement of a $\delta(\Lambda, s)$-closed set is said to be $\delta(\Lambda, s)$*-open* [10]. The union of all $\delta(\Lambda, s)$-open sets contained in A is called the $\delta(\Lambda, s)$*-interior* [10] of A and is denoted by $A_{\delta(\Lambda,s)}$. A subset A of a topological space (X, τ) is said to be $\delta p(\Lambda, s)$-*open* [10] if $A \subseteq [A^{(\Lambda,s)}]_{\delta(\Lambda,s)}$. The complement of a $\delta p(\Lambda, s)$-open set is said to be $\delta p(\Lambda, s)$-*closed* [10]. The family of all $\delta p(\Lambda, s)$-open (resp. $\delta p(\Lambda, s)$-closed) sets in a topological space (X, τ) is denoted by $\delta p(\Lambda, s)O(X, \tau)$ (resp. $\delta p(\Lambda, s)C(X, \tau)$). Let A be a subset of a topological space (X, τ). The intersection of all $\delta p(\Lambda, s)$-closed sets containing A is called *the* $\delta p(\Lambda, s)$-*closure* [10] of A and is denoted by $A^{\delta p(\Lambda,s)}$.

Lemma 2.1. [10] *For the $\delta p(\Lambda, s)$-closure of subsets A, B in a topological space (X, τ), the following properties hold:*

(1) If $A \subseteq B$, then $A^{\delta p(\Lambda,s)} \subseteq B^{\delta p(\Lambda,s)}$.

(2) A is $\delta p(\Lambda, s)$-closed in (X, τ) if and only if $A = A^{\delta p(\Lambda,s)}$.

(3) $A^{\delta p(\Lambda,s)}$ is $\delta p(\Lambda, s)$-closed, that is, $A^{\delta p(\Lambda,s)} = [A^{\delta p(\Lambda,s)}]^{\delta p(\Lambda,s)}$.

(4) $x \in A^{\delta p(\Lambda,s)}$ if and only if $A \cap V \neq \emptyset$ for every $V \in \delta p(\Lambda, s)O(X, \tau)$ containing x.

3 Sober $\delta p(\Lambda, s)$-R_0 spaces

In this section, we introduce the concept of sober $\delta p(\Lambda, s)$-R_0 spaces. Moreover, we characterize sober $\delta p(\Lambda, s)$-R_0 spaces.

Definition 3.1. *Let A be a subset of a topological space (X, τ). The $\delta p(\Lambda, s)$-kernel of A, denoted by $\delta p(\Lambda, s)Ker(A)$, is defined to be the set*

$$\delta p(\Lambda, s)Ker(A) = \cap\{U \in \delta p(\Lambda, s)O(X, \tau) \mid A \subseteq U\}.$$

Definition 3.2. [10] *A subset N_x of a topological space (X, τ) is called a $\delta p(\Lambda, s)$-neighborhood of a point $x \in X$ if there exists a $\delta p(\Lambda, s)$-open set U such that $x \in U \subseteq N_x$.*

Lemma 3.3. *Let A be a subset of a topological space (X, τ) and $x \in X$. Then $\delta p(\Lambda, s)Ker(A) = \{x \in X \mid \{x\}^{\delta p(\Lambda,s)} \cap A \neq \emptyset\}$.*

Proof. Let $x \in \delta p(\Lambda, s)Ker(A)$ and suppose that $\{x\}^{\delta p(\Lambda,s)} \cap A = \emptyset$. Thus $x \notin X - \{x\}^{\delta p(\Lambda,s)}$ which is a $\delta p(\Lambda, s)$-open set containing A. This is absurd, since $x \in \delta p(\Lambda, s)Ker(A)$. Hence $\{x\}^{\delta p(\Lambda,s)} \cap A \neq \emptyset$. Next, let x be such that $\{x\}^{\delta p(\Lambda,s)} \cap A \neq \emptyset$ and suppose that $x \notin \delta p(\Lambda, s)Ker(A)$. Then there exists a $\delta p(\Lambda, s)$-open set U containing A and $x \notin U$. Let $y \in \{x\}^{\delta p(\Lambda,s)} \cap A$. Thus U is a $\delta p(\Lambda, s)$-neighborhood of y which does not contain x. This contradiction leads to $x \in \delta p(\Lambda, s)Ker(A)$ and so the claim follows. ☐

Definition 3.4. *A topological space* (X, τ) *is said to be sober* $\delta p(\Lambda, s)$*-*R_0 *if* $\bigcap_{x\in X}\{x\}^{\delta p(\Lambda,s)} = \emptyset$.

Theorem 3.5. *A topological space* (X, τ) *is sober* $\delta p(\Lambda, s)$*-*R_0 *if and only if* $\delta p(\Lambda, s)Ker(\{x\}) \neq X$ *for each* $x \in X$.

Proof. Suppose that the space (X, τ) is sober $\delta p(\Lambda, s)$-R_0. Assume that there is a point y in X such that $\delta p(\Lambda, s)Ker(\{y\}) = X$. Let x be any point of X. Then $x \in V$ for every $\delta p(\Lambda, s)$-open set V containing y and hence $y \in \{x\}^{\delta p(\Lambda,s)}$ for each $x \in X$. Thus $y \in \bigcap_{x\in X}\{x\}^{\delta p(\Lambda,s)}$. This is a contradiction.

Conversely, assume that $\delta p(\Lambda, s)Ker(\{x\}) \neq X$ for each $x \in X$. If there exists a point y in X such that $y \in \bigcap_{x\in X}\{x\}^{\delta p(\Lambda,s)}$, then every $\delta p(\Lambda, s)$-open set containing y must contain every point of X. This implies that the space (X, τ) is the unique $\delta p(\Lambda, s)$-open set containing y. Thus, $\delta p(\Lambda, s)Ker(\{x\}) = X$ which is a contradiction. This shows that (X, τ) is sober $\delta p(\Lambda, s)$-R_0. □

Definition 3.6. *A function* $f : (X, \tau) \to (Y, \sigma)$ *is called* $\delta p(\Lambda, s)$*-closed if* $f(F)$ *is* $\delta p(\Lambda, s)$*-closed in* Y *for every* $\delta p(\Lambda, s)$*-closed set* F *of* X.

Theorem 3.7. *If* $f : (X, \tau) \to (Y, \sigma)$ *is an injective* $\delta p(\Lambda, s)$*-closed function and* (X, τ) *is sober* $\delta p(\Lambda, s)$*-*R_0*, then* (Y, σ) *is sober* $\delta p(\Lambda, s)$*-*R_0.

Proof. Since (X, τ) is sober $\delta p(\Lambda, s)$-R_0, $\bigcap_{x\in X}\{x\}^{\delta p(\Lambda,s)} = \emptyset$. Since f is a $\delta p(\Lambda, s)$-closed injection, we have

$$\begin{aligned}\emptyset = f(\bigcap_{x\in X}\{x\}^{\delta p(\Lambda,s)}) &= \bigcap_{x\in X} f(\{x\}^{\delta p(\Lambda,s)})\\ &\supseteq \bigcap_{x\in X}\{f(x)\}^{\delta p(\Lambda,s)} \supseteq \bigcap_{y\in Y}\{y\}^{\delta p(\Lambda,s)}.\end{aligned}$$

Thus, (Y, σ) is sober $\delta p(\Lambda, s)$-R_0. □

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References

[1] C. Boonpok, C. Viriyapong, On some forms of closed sets and related topics, Eur. J. Pure Appl. Math., **16,** no. 1, (2023), 336–362.

[2] M. Caldas, T. Fukutake, S. Jafari, T. Noiri, Some applications of δ-preopen sets in topological spaces, Bull. Inst. Math. Acad. Sinica, **33,** no. 3, (2005), 261–276.

[3] M. Caldas, M. Ganster, D. N. Georgiou, S. Jafari, T. Noiri, δ-semiopen sets in topological spaces, Topology Proc., **29,** no. 2, (2005), 369–383.

[4] M. Caldas, D. N. Georgiou, S. Jafari, T. Noiri, More on δ-semiopen sets, Note di Matematica, **22,** no. 2, (2003), 1–14.

[5] M. Caldas, J. Dontchev, G.Λ_s-sets and g.V_s-sets, arXiv:math/9810080v1 [math.GN], (1998).

[6] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, **70,** (1963), 36–41.

[7] A. S. Mashhour, M. E. Abd El-Monsef, S. N. El-Deeb, On precontinuous and weak precontinuous mappings, Proc. Math. Phys. Soc. Egypt, **53,** (1982), 47–53.

[8] J. H. Park, B. Y. Lee, M. J. Son, On δ-semiopen sets in topological spaces, J. Indian Acad. Math., **19,** (1997), 59–67.

[9] S. Raychaudhuri, M. N. Mukherjee, On δ-almost continuity and δ-preopen sets, Bull. Inst. Math. Acad. Sinica, **21,** (1993), 357–366.

[10] N. Srisarakham, C. Boonpok, On characterizations of $\delta p(\Lambda, s)$-$\mathscr{D}_1$ spaces, Int. J. Math. Comput. Sci., **18,** no. 4, (2023), 743–747.

[11] N. V. Veličko, H-Closed Topological Spaces, Trans. Amer. Math. Soc., **78,** no. 2, (1968), 103–118.