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On $S\Lambda_s$-regular spaces and $S\Lambda_s$-normal spaces

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Abstract

In this paper, we deal with the concepts of $S\Lambda_s$-regular spaces and $S\Lambda_s$-normal spaces. Moreover, we investigate several characterizations of $S\Lambda_s$-regular spaces and $S\Lambda_s$-normal spaces.

1 Introduction

Singal and Arya [8] defined a new separation axiom called almost regularity which is weaker than regularity. El-Deeb et al. [4] introduced and studied the notion of p-regular spaces. Malghan and Navalagi [5] introduced and investigated the concept of almost p-regular spaces as a generalization of p-regularity. Noiri [6] defined a new class of sets called rgp-closed sets and investigated some properties of almost p-regular spaces by utilizing rgp-closed sets. Ekici [3] introduced a new class of spaces, called γ-normal spaces and investigated the relationships among s-normal spaces, p-normal spaces and γ-normal spaces. Ekici and Noiri [2] introduced and studied the notions of δp-normal spaces, almost δp-normal spaces and mildly δp-normal spaces. Torton et al. [11] introduced and investigated the concepts of $\mu_{(m,n)}$-regular

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spaces and $\mu_{(m,n)}$-normal spaces. In [1], the present authors introduced the concept of $s(\Lambda, s)$-open sets. Pue-on and Boonpok [7] introduced and studied the concepts of $\delta s(\Lambda, s)$-closed sets and $\delta s(\Lambda, s)$-open sets. In this paper, we introduce the notions of $S\Lambda_s$-regular spaces and $S\Lambda_s$-normal spaces. Furthermore, several properties of $S\Lambda_s$-regular spaces and $S\Lambda_s$-normal spaces are discussed.

2 Preliminaries

Let A be a subset of a topological space (X, τ). A subset A is called $s(\Lambda, s)$-*open* [1] if $A \subseteq [A_{(\Lambda,s)}]^{(\Lambda,s)}$. The family of all $s(\Lambda, s)$-open sets in a topological space (X, τ) is denoted by $s(\Lambda, s)O(X, \tau)$. The complement of a $s(\Lambda, s)$-open set is called $s(\Lambda, s)$-*closed*. The intersection of all $s(\Lambda, s)$-closed sets containing A is called the $s(\Lambda, s)$-*closure* of A and is denoted by $A^{s(\Lambda,s)}$. A subset A is called $s(\Lambda, s)$-*regular* if A is $s(\Lambda, s)$-open and $s(\Lambda, s)$-closed. The family of all $s(\Lambda, s)$-regular sets in a topological space (X, τ) is denoted by $s(\Lambda, s)r(X, \tau)$. A point x of X is called a $\delta(\Lambda, s)$-*cluster point* [10] of A if $A \cap [U^{(\Lambda,s)}]_{(\Lambda,s)} \neq \emptyset$ for every (Λ, s)-open set U of X containing x. The set of all $\delta(\Lambda, s)$-cluster points of A is called the $\delta(\Lambda, s)$-*closure* [10] of A and is denoted by $A^{\delta(\Lambda,s)}$. A subset A is called $\delta(\Lambda, s)$-*closed* [10] if $A = A^{\delta(\Lambda,s)}$. The complement of a $\delta(\Lambda, s)$-closed set is said to be $\delta(\Lambda, s)$-*open*. A subset A is called $\delta s(\Lambda, s)$-*open* [7] if $A \subseteq [A_{(\Lambda,s)}]^{\delta(\Lambda,s)}$. The complement of a $\delta s(\Lambda, s)$-open set is called $\delta s(\Lambda, s)$-*closed*. The family of all $\delta s(\Lambda, s)$-open sets in a topological space (X, τ) is denoted by $\delta s(\Lambda, s)O(X, \tau)$. A point x of X is called a $\delta s(\Lambda, s)$-*cluster point* [7] of A if $A \cap U \neq \emptyset$ for every $\delta s(\Lambda, s)$-open set U of X containing x. The set of all $\delta s(\Lambda, s)$-cluster points of A is called the $\delta s(\Lambda, s)$-*closure* [7] of A and is denoted by $A^{\delta s(\Lambda,s)}$.

3 Characterizations of $S\Lambda_s$-regular spaces and $S\Lambda_s$-normal spaces

In this section, we introduce the notions of $S\Lambda_s$-regular spaces and $S\Lambda_s$-normal spaces. Moreover, some characterizations of $S\Lambda_s$-regular spaces and $S\Lambda_s$-normal spaces are discussed.

Definition 3.1. *A topological space (X, τ) is said to be $S\Lambda_s$-regular if for each $s(\Lambda, s)$-closed set F of X and each point $x \notin F$, there exist $s(\Lambda, s)$-open sets U and V such that $x \in U$, $F \subseteq V$ and $U \cap V = \emptyset$.*

Lemma 3.2. [9] *For a subset A of a topological space (X, τ), the following properties hold:*

(1) If A is a $s(\Lambda, s)$-regular set, then it is $\delta s(\Lambda, s)$-open.

(2) If A is a $\delta s(\Lambda, s)$-open set, then it is $s(\Lambda, s)$-open.

(3) If A is a $s(\Lambda, s)$-open set, then $A^{s(\Lambda,s)}$ is $s(\Lambda, s)$-regular.

Theorem 3.3. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is $S\Lambda_s$-regular.

(2) For each $s(\Lambda, s)$-closed set F and each point $x \notin F$, there exist $U, V \in \delta s(\Lambda, s)O(X, \tau)$ such that $x \in U$, $F \subseteq V$ and $U \cap V = \emptyset$.

(3) For each point $x \in X$ and each $s(\Lambda, s)$-open set V containing x, there exists $U \in \delta s(\Lambda, s)O(X, \tau)$ such that $x \in U \subseteq U^{\delta s(\Lambda,s)} \subseteq V$.

Proof. (1) $\Rightarrow$ (2): Let F be a $s(\Lambda, s)$-closed set and $x \notin F$. Then, there exist $G, H \in s(\Lambda, s)O(X, \tau)$ such that $x \in G$, $F \subseteq H$ and $G \cap H = \emptyset$. By Lemma 3.2, $G^{s(\Lambda,s)}$ is $s(\Lambda, s)$-regular and $G^{s(\Lambda,s)} \cap H = \emptyset$. Thus, $G^{s(\Lambda,s)} \cap H^{s(\Lambda,s)} = \emptyset$. Now, we put $U = G^{s(\Lambda,s)}$ and $V = H^{s(\Lambda,s)}$, then U and V are $\delta s(\Lambda, s)$-open sets such that $x \in U$, $F \subseteq V$ and $U \cap V = \emptyset$.

(2) $\Rightarrow$ (3): Let $x \in X$ and V be any $s(\Lambda, s)$-open set containing x. Since $x \notin X - V$, there exist $U, G \in \delta s(\Lambda, s)O(X, \tau)$ such that $x \in U$, $X - V \subseteq G$ and $U \cap G = \emptyset$. Since $X - G$ is $\delta s(\Lambda, s)$-closed and $U \subseteq X - G$, $x \in U \subseteq U^{\delta s(\Lambda,s)} \subseteq X - G \subseteq V$.

(3) $\Rightarrow$ (1): Let F be a $s(\Lambda, s)$-closed set and $x \notin F$. Then, $X - F$ is $s(\Lambda, s)$-open set containing x. By (3), there exists $U \in \delta s(\Lambda, s)O(X, \tau)$ such that $x \in U \subseteq U^{\delta s(\Lambda,s)} \subseteq X - F$. Thus, $x \in U$, $F \subseteq X - U^{\delta s(\Lambda,s)}$ and $U \cap (X - U^{\delta s(\Lambda,s)}) = \emptyset$. Since $\delta s(\Lambda, s)O(X, \tau) \subseteq s(\Lambda, s)O(X, \tau)$, (X, τ) is $S\Lambda_s$-regular. □

Definition 3.4. *A topological space (X, τ) is said to be $S\Lambda_s$-normal if for each disjoint $s(\Lambda, s)$-closed sets F and K of X, there exist $s(\Lambda, s)$-open sets U and V such that $F \subseteq U$, $K \subseteq V$ and $U \cap V = \emptyset$.*

Theorem 3.5. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is $S\Lambda_s$-normal.

(2) For each disjoint $s(\Lambda, s)$-closed sets F and K of X, there exist $U, V \in \delta s(\Lambda, s)O(X, \tau)$ such that $F \subseteq U$, $K \subseteq V$ and $U \cap V = \emptyset$.

(3) For each $s(\Lambda, s)$-closed set F and each $s(\Lambda, s)$-open set V containing F, there exists $U \in \delta s(\Lambda, s)O(X, \tau)$ such that $F \subseteq U \subseteq U^{\delta s(\Lambda,s)} \subseteq V$.

Proof. The proof is analogous to that of Theorem 3.3. □

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