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Absolute $g^*\omega\alpha$-Continuous Function in Bigeneralized Topological Spaces

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Abstract

In this paper, absolute $g^*\omega\alpha$-continuous functions in bigeneralized topological spaces is introduced and characterized.

1 Introduction

In 2011, Boonpok [5] introduced the concept of bigeneralized topological space (briefly BGTS). On the other hand, in 2015, Benchalli et al. [3] introduced the notion of generalized star $\omega\alpha$-sets (briefly $g^*\omega\alpha$-sets) in topological spaces. In this paper, the author defines and introduces absolute $g^*\omega\alpha$-continuity in BGTS. Characterizations and properties of this newly defined function are also explored.

To establish a common understanding of terminologies and notations in topology, the reader may utilize standard conventions defined by Dugundji [6]. Also, the next statements are some additional basic concepts necessary to define absolute $g^*\omega\alpha$-continuous functions in BGTS.

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Let X be a nonempty set. A subset μ of $\mathscr{P}(X)$ is said to be a generalized topology (briefly GT) on X if $\varnothing \in \mu$ and the arbitrary union of elements of μ belongs to μ. If μ is a GT on X, then (X, μ) is said to be a *generalized topological space* (briefly GTS), and the elements of μ are called *μ-open sets*. The complement of a μ-open set is called *μ-closed set*. If $A \subseteq X$, then the *μ-closure of* A, denoted by $c_\mu(A)$, is the intersection of all μ-closed sets containing A. The *μ-interior of* A, denoted by $i_\mu(A)$, is the union of all μ-open sets contained in A.

In 2009, Benchalli et al. [1] introduced the following definitions:
A set A of a GTS (X, μ) is said to be μ-α-closed if $c_\mu(i_\mu(c_\mu(A)) \subseteq A$ and μ-$\omega\alpha$-closed if $\alpha c_\mu(A) \subseteq U$ whenever $A \subseteq U$ and U μ-ω-open in X. The complement of a μ-$\omega\alpha$-closed set is *μ-$\omega\alpha$-open* set.

A subset A of X is said to be *μ-generalized star $\omega\alpha$-closed* (briefly μ-$g^*\omega\alpha$-closed) set if $c_\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is μ-$\omega\alpha$-open in X. The complement of μ-$g^*\omega\alpha$-closed set is said to be *μ-$g^*\omega\alpha$-open set*. If A is both μ-$g^*\omega\alpha$-closed set and μ-$g^*\omega\alpha$-open set, then A is said to be *μ-$g^*\omega\alpha$-clopen set*. The union of all the μ-$g^*\omega\alpha$-open sets contained in A is called the *μ-$g^*\omega\alpha$-interior of* A, denoted by $g^*\omega\alpha i_\mu(A)$. The intersection of all the μ-$g^*\omega\alpha$-closed sets containing A is called the *μ-$g^*\omega\alpha$-closure of* A denoted by $g^*\omega\alpha c_\mu(A)$.

If μ_1 and μ_2 are generalized topologies on X, then the triple (X, μ_1, μ_2) is said to be a *bigeneralized topological space* (briefly BGTS). Throughout this paper, m and n take values from the set $\{1, 2\}$ where $m \neq n$.

The following definition is due to Boonpok et al. [4].

Definition 1.1. [4] Let $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ be a function. Then f is *$\mu^{(m,n)}$-continuous at a point* $x \in X$ if for each μ_Y^m-open set V containing $f(x)$, there exists a μ_X^n-open set U containing x such that $f(U) \subseteq V$. If f is $\mu^{(m,n)}$-continuous at every point $x \in X$, then f is *$\mu^{(m,n)}$-continuous.*

The following results and definitions are introduced by Nalzaro et al. [7].

Definition 1.2. A function $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ is said to be *$\mu^{(m,n)}$-$g^*\omega\alpha$ continuous at a point* $x \in X$ if for each μ_Y^m-open set V containing $f(x)$, there exists a μ_X^n-$g^*\omega\alpha$ open set U containing x such that $f(U) \subseteq V$. If f is $\mu^{(m,n)}$-$g^*\omega\alpha$ continuous at every point $x \in X$. *$\mu^{(m,n)}$-$g^*\omega\alpha$ continuous*

Lemma 1.3. Let (X, μ) be a GTS and $y \in X$. $y \in g^*\omega\alpha c_\mu(A)$ if and only if for every μ-$g^*\omega\alpha$ open set U with $y \in U$, $U \cap A \neq \varnothing$;

Corollary 1.4. Every μ-open set is μ-$g^*\omega\alpha$-open.

2 Main Results

Definition 2.1. A function $f : (X, \mu_X^1, \mu_X^2) \rightarrow (Y, \mu_Y^1, \mu_Y^2)$ is said to be *absolute $\mu^{(m,n)}$-$g^*\omega\alpha$ continuous* if for each μ_Y^m-$g^*\omega\alpha$ open set U in Y, $f^{-1}(U)$ is μ_X^n-$g^*\omega\alpha$ open in X.

Example 2.2. Consider the sets $X = \{a, b, c\}$ and $Y = \{d, e\}$ with GT's $\mu_X^1 = \{\varnothing, \{a, b\}, \{a, c\}, X\}$, $\mu_X^2 = \{\varnothing, \{a, c\}, \{b, c\}, X\}$, $\mu_Y^1 = \{\varnothing, \{d\}, Y\}$, and $\mu_Y^2 = \{\varnothing, \{e\}, Y\}$. Then the μ_X^1-$g^*\omega\alpha$ open sets in X are $\varnothing$, X, $\{a\}$, $\{a, b\}$ and $\{a, c\}$ and μ_X^2-$g^*\omega\alpha$ open sets are $\varnothing$, X, $\{b, c\}$, $\{a, c\}$ and $\{c\}$. Also, the μ_Y^1-$g^*\omega\alpha$ open sets in X are $\varnothing$, Y, and $\{d\}$ and the μ_Y^2-$g^*\omega\alpha$ open sets are $\varnothing$, Y, and $\{e\}$. Let $f : (Y, \mu_Y^1, \mu_Y^2) \rightarrow (X, \mu_X^1, \mu_X^2)$ be defined by $f(a) = e$ and $f(b) = d = f(c)$. Then f is absolute $\mu^{(1,2)}$-$g^*\omega\alpha$ continuous since $f^{-1}(\varnothing) = \varnothing$, $f^{-1}(\{d\}) = \{b, c\}$, and $f^{-1}(Y) = X$ are μ_X^2-$g^*\omega\alpha$ open sets in X where $\varnothing$, $\{d\}$, and Y are μ_Y^1-$g^*\omega\alpha$ open sets in X. Also, f is absolute $\mu^{(2,1)}$-$g^*\omega\alpha$ continuous.

Lemma 2.3. Let $f : (X, \mu_X^1, \mu_X^2) \rightarrow (Y, \mu_Y^1, \mu_Y^2)$ be a function. If for each μ_Y^m-open set U of Y, $f^{-1}(U)$ is μ_X^n-$g^*\omega\alpha$ open in X, then f is $\mu^{(m,n)}$-$g^*\omega\alpha$ continuous.

Proof. Let $x \in X$ and V be any μ_Y^m-open set in Y such that $f(x) \in V$. By assumption, $f^{-1}(V)$ is μ_X^n-$g^*\omega\alpha$ open in X with $x \in f^{-1}(V)$. Take $O = f^{-1}(V)$. Then $x \in O$ and $f(O) \subseteq V$. Therefore, f is $\mu^{(m,n)}$-$g^*\omega\alpha$ continuous. □

Theorem 2.4. If $f : (X, \mu_X^1, \mu_X^2) \rightarrow (Y, \mu_Y^1, \mu_Y^2)$ is an absolute $\mu^{(m,n)}$-$g^*\omega\alpha$ continuous function, then f is $\mu^{(m,n)}$-$g^*\omega\alpha$ continuous.

Proof. Let V be any μ_Y^m-open set in Y. Then by Corollary 1.4, V is μ_Y^m-$g^*\omega\alpha$ open in Y. Since f is absolute $\mu^{(m,n)}$-$g^*\omega\alpha$ continuous, $f^{-1}(V)$ is μ_X^n-$g^*\omega\alpha$ open in X. Therefore, by Lemma 2.3, f is $\mu^{(m,n)}$-$g^*\omega\alpha$ continuous. □

Theorem 2.5. A function $f : (X, \mu_X^1, \mu_X^2) \rightarrow (Y, \mu_Y^1, \mu_Y^2)$ is absolute $\mu^{(m,n)}$-$g^*\omega\alpha$ continuous if and only if $f^{-1}(U)$ is μ_X^n-$g^*\omega\alpha$ closed in X for every μ_Y^m-$g^*\omega\alpha$ closed set U in Y.

Proof. Suppose that f is absolute $\mu^{(m,n)}$-$g^*\omega\alpha$ continuous. Let U be a μ_Y^m-$g^*\omega\alpha$ closed in Y. Then $Y \setminus U$ is μ_Y^m-$g^*\omega\alpha$ open in Y. Hence, $f^{-1}(Y \setminus U) = X \setminus f^{-1}(U)$ is μ_X^n-$g^*\omega\alpha$ open in X. Thus, $f^{-1}(U)$ is μ_X^n-$g^*\omega\alpha$ closed in X.

Conversely, let O be a μ_Y^m-$g^*\omega\alpha$ open set in Y. Then $Y \setminus O$ is μ_Y^m-$g^*\omega\alpha$ closed in Y. By assumption, $f^{-1}(Y \setminus O)= X \setminus f^{-1}(O)$ is μ_X^n-$g^*\omega\alpha$ closed in X. Therefore, $f^{-1}(O)$ is μ_X^n-$g^*\omega\alpha$ open in X. □

Theorem 2.6. Let $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ be an absolute $\mu^{(m,n)}$-$g^*\omega\alpha$ continuous function. Then the following hold:

(i) For each $x \in X$ and for every μ_Y^m-$g^*\omega\alpha$ open set V in Y containing $f(x)$, there exists a μ_X^n-$g^*\omega\alpha$ open set U containing x such that $f(U) \subseteq V$;

(ii) $f(g^*\omega\alpha c_{\mu_X^n}(A)) \subseteq g^*\omega\alpha c_{\mu_Y^m}(f(A))$ for every $A \subseteq X$;

(iii) $g^*\omega\alpha c_{\mu_X^n}(f^{-1}(B)) \subseteq f^{-1}(g^*\omega\alpha c_{\mu_Y^m}(B))$ for every $B \subseteq Y$.

Proof. Let $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ be an absolute $\mu^{(m,n)}$-$g^*\omega\alpha$ continuous function.
(i) Let $x \in X$ and V be a μ_Y^m-$g^*\omega\alpha$ open set in Y with $f(x) \in V$. Then $Y \setminus V$ is μ_Y^m-$g^*\omega\alpha$ closed in Y. By Theorem 2.5, $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is a μ_X^n-$g^*\omega\alpha$ closed set in X. Hence, $f^{-1}(V)$ is a μ_X^n-$g^*\omega\alpha$ open set in X. Let $U = f^{-1}(V)$. Then $x \in U$ and $f(U) \subseteq V$. Thus, (i) holds.

(ii) Let $A \subseteq X$ and $x \in g^*\omega\alpha c_{\mu_X^n}(A)$. Then $f(x) \in f(g^*\omega\alpha c_{\mu_X^n}(A))$. Let V be a μ_Y^m-$g^*\omega\alpha$ open set in Y with $f(x) \in V$. By (i), there exists a μ_X^n-$g^*\omega\alpha$ open set U with $x \in U$ and $f(U) \subseteq V$. Since $x \in g^*\omega\alpha c_{\mu_X^n}(A)$,by Lemma 1.3, $A \cap U \neq \varnothing$. It follows that $\varnothing \neq f(A \cap U) \subseteq f(A) \cap f(U) \subseteq f(A) \cap V$. Hence, $f(A) \cap V \neq \varnothing$. Therefore, by Lemma 1.3, $f(x) \in g^*\omega\alpha c_{\mu_Y^m}(f(A))$. Thus, (ii) holds.

(iii) Let $B \subseteq Y$. Take A=$f^{-1}(B)$ in (ii). Then

$$f(g^*\omega\alpha c_{\mu_X^n}(f^{-1}(B))) \subseteq g^*\omega\alpha c_{\mu_Y^m}(f(f^{-1}(B))) \subseteq g^*\omega\alpha c_{\mu_Y^m}(B).$$

Therefore, (iii) holds. □

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