International Journal of Mathematics and
Computer Science, **19**(2024), no. 1, 121–124

On characterizations of $S\Lambda_s$-T_2 spaces

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(Received May 18, 2023, Accepted July 20, 2023,
Published August 31, 2023)

Abstract

Our main purpose is to introduce the concept of $S\Lambda_s$-T_2 spaces. Moreover, we investigate some characterizations of $S\Lambda_s$-T_2 spaces.

1 Introduction

Veličko [9] introduced δ-open sets, which are stronger than open sets. Park et al. [5] have offered a new notion called δ-semiopen sets which are stronger than semi-open sets but weaker than δ-open sets and investigated the relationships among several types of these open sets. Caldas et al. [4] investigated some weak separation axioms by utilizing δ-semiopen sets and the δ-semiclosure operator. Caldas et al. [3] investigated the notion of δ-Λ_s-semiclosed sets which is defined as the intersection of a δ-Λ_s-set and a δ-semiclosed set. In [1], the present authors introduced and investigated the concept of (Λ, s)-closed sets by utilizing the notions of Λ_s-sets and semi-closed sets. Pue-on and Boonpok [6] introduced and investigated the notions of $\delta s(\Lambda, s)$-open sets and $\delta s(\Lambda, s)$-closed sets. Srisarakham and Boonpok [8] introduced and studied the concept of $\delta p(\Lambda, s)$-$\mathscr{D}_1$ spaces. Buadong et al.

Key words and phrases: $\delta(\Lambda, s)$-open set, $S\Lambda_s$-T_2 space.
Corresponding author: Montri Thongmoon.
AMS (MOS) Subject Classifications: 54A05, 54D10.
ISSN 1814-0432, 2024, http://ijmcs.future-in-tech.net

[2] investigated some characterizations of T_1-GTMS spaces and T_2-GTMS spaces. In this paper, we introduce the concept of $S\Lambda_s$-T_2 spaces. Moreover, we discuss several characterizations of $S\Lambda_s$-T_2 spaces.

2 Preliminaries

A subset A of a topological space (X, τ) is called (Λ, s)-*closed* [1] if $A = T \cap C$, where T is a Λ_s-set and C is a semi-closed set. The complement of a (Λ, s)-closed set is called (Λ, s)-*open*. A subset A of a topological space (X, τ) is called $s(\Lambda, s)$-*open* [1] if $A \subseteq [A_{(\Lambda,s)}]^{(\Lambda,s)}$. The family of all $s(\Lambda, s)$-open sets in a topological space (X, τ) is denoted by $s(\Lambda, s)O(X, \tau)$. The complement of a $s(\Lambda, s)$-open set is called $s(\Lambda, s)$-*closed*. Let A be a subset of a topological space (X, τ). The intersection of all $s(\Lambda, s)$-closed sets containing A is called the $s(\Lambda, s)$-*closure* [7] of A and is denoted by $A^{s(\Lambda,s)}$. A subset A is said to be $s(\Lambda, s)$-*regular* [7] if A is $s(\Lambda, s)$-open and $s(\Lambda, s)$-closed. The family of all $s(\Lambda, s)$-regular sets in a topological space (X, τ) is denoted by $s(\Lambda, s)r(X, \tau)$. A point x of X is called a $\delta(\Lambda, s)$-*cluster point* [8] of A if $A \cap [U^{(\Lambda,s)}]_{(\Lambda,s)} \neq \emptyset$ for every (Λ, s)-open set U of X containing x. The set of all $\delta(\Lambda, s)$-cluster points of A is called the $\delta(\Lambda, s)$-*closure* [8] of A and is denoted by $A^{\delta(\Lambda,s)}$. A subset A is called $\delta(\Lambda, s)$-*closed* [8] if $A = A^{\delta(\Lambda,s)}$. The complement of a $\delta(\Lambda, s)$-closed set is said to be $\delta(\Lambda, s)$-*open*. A subset A is called $\delta s(\Lambda, s)$-*open* [6] if $A \subseteq [A_{(\Lambda,s)}]^{\delta(\Lambda,s)}$. The complement of a $\delta s(\Lambda, s)$-open set is called $\delta s(\Lambda, s)$-*closed*. The family of all $\delta s(\Lambda, s)$-open sets in a topological space (X, τ) is denoted by $\delta s(\Lambda, s)O(X, \tau)$. A point x of X is called a $\delta s(\Lambda, s)$-*cluster point* [6] of A if $A \cap U \neq \emptyset$ for every $\delta s(\Lambda, s)$-open set U of X containing x. The set of all $\delta s(\Lambda, s)$-cluster points of A is called the $\delta s(\Lambda, s)$-*closure* [6] of A and is denoted by $A^{\delta s(\Lambda,s)}$.

3 Characterizations of $S\Lambda_s$-T_2 spaces

In this section, we introduce the concept of $S\Lambda_s$-T_2 spaces. Moreover, we discuss some characterizations of $S\Lambda_s$-T_2 spaces.

Definition 3.1. *A topological space (X, τ) is said to be $S\Lambda_s$-T_2 if, for each pair of distinct points $x, y \in X$, there exist $U, V \in s(\Lambda, s)O(X, \tau)$ such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.*

Lemma 3.2. [7] *For a subset A of a topological space (X, τ), the following properties hold:*

(1) *If A is a $s(\Lambda, s)$-regular set, then it is $\delta s(\Lambda, s)$-open.*

(2) *If A is a $\delta s(\Lambda, s)$-open set, then it is $s(\Lambda, s)$-open.*

(3) *If A is a $s(\Lambda, s)$-open set, then $A^{s(\Lambda,s)}$ is $s(\Lambda, s)$-regular.*

Theorem 3.3. *For a topological space (X, τ), the following properties are equivalent:*

(1) *(X, τ) is $S\Lambda_s$-T_2;*

(2) *For each pair of distinct points x, y of X, there exist $U, V \in s(\Lambda, s)r(X, \tau)$ such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.*

(3) *For each pair of distinct points x, y of X, there exist $U, V \in \delta s(\Lambda, s)O(X, \tau)$ such that $x \in U$, $y \in V$ and $U^{\delta s(\Lambda,s)} \cap V^{\delta s(\Lambda,s)} = \emptyset$.*

(4) *For each pair of distinct points x, y of X, there exist $U, V \in \delta s(\Lambda, s)O(X, \tau)$ such that $x \in U$, $y \in V$ and $U^{s(\Lambda,s)} \cap V^{s(\Lambda,s)} = \emptyset$.*

(5) *For each pair of distinct points x, y of X, there exist $U, V \in \delta s(\Lambda, s)O(X, \tau)$ such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.*

Proof. (1) $\Rightarrow$ (2): Suppose that (X, τ) is $S\Lambda_s$-T_2. Then, for each pair of distinct points x, y of X, there exist $G, H \in s(\Lambda, s)O(X, \tau)$ such that $x \in G$, $y \in H$ and $G \cap H = \emptyset$. Thus, $G^{s(\Lambda,s)} \cap H = \emptyset$. By Lemma 3.2, we have $G^{s(\Lambda,s)} \in s(\Lambda, s)r(X, \tau)$ and $G^{s(\Lambda,s)} \cap H^{s(\Lambda,s)} = \emptyset$. Now set $U = G^{s(\Lambda,s)}$ and $V = H^{s(\Lambda,s)}$. Then, U and V are $s(\Lambda, s)$-regular sets such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.

(2) $\Rightarrow$ (3): This follows from the facts that $s(\Lambda, s)r(X, \tau) \subseteq \delta s(\Lambda, s)O(X, \tau)$ and $U^{\delta s(\Lambda,s)} = U^{s(\Lambda,s)} = U$ for every $U \in s(\Lambda, s)r(X, \tau)$.

(3) $\Rightarrow$ (4): This follows from the fact that $U^{\delta s(\Lambda,s)} = U^{s(\Lambda,s)}$ for every $U \in \delta s(\Lambda, s)O(X, \tau)$.

(4) $\Rightarrow$ (5): This is obvious.

(5) $\Rightarrow$ (1): This is obvious since $\delta s(\Lambda, s)O(X, \tau) \subseteq s(\Lambda, s)O(X, \tau)$. □

Definition 3.4. *A topological space (X, τ) is said to be $S\Lambda_s$-Urysohn if, for each pair of distinct points x, y of X, there exist $U, V \in s(\Lambda, s)O(X, \tau)$ such that $x \in U$, $y \in V$ and $U^{(\Lambda,s)} \cap V^{(\Lambda,s)} = \emptyset$.*

Theorem 3.5. *A topological space (X, τ) is $S\Lambda_s$-Urysohn if and only if for each pair of distinct points x, y of X, there exist $U, V \in \delta s(\Lambda, s)O(X, \tau)$ such that $x \in U$, $y \in V$ and $U^{(\Lambda,s)} \cap V^{(\Lambda,s)} = \emptyset$.*

Proof. Necessity. Suppose that (X,τ) is $S\Lambda_s$-Urysohn. Then, for each pair of distinct points x, y of X, there exist $U, V \in s(\Lambda,s)O(X,\tau)$ such that $x \in U, y \in V$ and $U^{(\Lambda,s)} \cap V^{(\Lambda,s)} = \emptyset$. Since $U \in s(\Lambda,s)O(X,\tau)$, $U^{(\Lambda,s)} = [U_{(\Lambda,s)}]^{(\Lambda,s)}$ and $U^{(\Lambda,s)}$ is $r(\Lambda,s)$-closed. Thus, $U^{(\Lambda,s)}, V^{(\Lambda,s)} \in s(\Lambda,s)r(X,\tau) \subseteq \delta s(\Lambda,s)O(X,\tau)$. It is obvious that $x \in U^{(\Lambda,s)}$, $y \in V^{(\Lambda,s)}$ and

$$[U^{(\Lambda,s)}]^{(\Lambda,s)} \cap [V^{(\Lambda,s)}]^{(\Lambda,s)} = U^{(\Lambda,s)} \cap V^{(\Lambda,s)} = \emptyset.$$

Sufficiency. The proof is obvious since $\delta s(\Lambda,s)O(X,\tau) \subseteq s(\Lambda,s)O(X,\tau)$. □

Acknowledgment. This research project was financially supported by Mahasarakham University.

References

[1] C. Boonpok, C. Viriyapong, On some forms of closed sets and related topics, Eur. J. Pure Appl. Math., **16,** no. 1, (2023), 336–362.

[2] S. Buadong, C. Viriyapong, C. Boonpok, On generalized topology and minimal structure spaces, Int. J. Math. Anal., **5,** no. 31, (2011), 1507–1516.

[3] M. Caldas, M. Ganster, D. N. Georgiou, S. Jafari, T. Noiri, δ-semiopen sets in topological spaces, Topology Proc., **29,** no. 2, (2005), 369–383.

[4] M. Caldas, D. N. Georgiou, S. Jafari, T. Noiri, More on δ-semiopen sets, Note di Matematica, **22,** no. 2, (2003), 1–14.

[5] J. H. Park, B. Y. Lee, M. J. Son, On δ-semiopen sets in topological spaces, J. Indian Acad. Math., **19,** (1997), 59–67.

[6] P. Pue-on, C. Boonpok, On $\delta s(\Lambda,s)$-open sets in topological spaces, Int. J. Math. Comput. Sci., **18,** no. 4, (2023), 749–753.

[7] N. Srisarakham, C. Boonpok, Some properties of $S\Lambda_s$-closed spaces, Int. J. Math. Comput. Sci., **19,** no. 1, (2024), 117–120.

[8] N. Srisarakham, C. Boonpok, On characterizations of $\delta p(\Lambda,s)$-$\mathscr{D}_1$ spaces, Int. J. Math. Comput. Sci., **18,** no. 4, (2023), 743–747.

[9] N. V. Veličko, H-closed topological spaces, Amer. Math. Soc. Transl., **78,** no. 2, (1968), 102–118.