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Numerical simulation of new solitons-solutions having anti-cubic and variable perturbation on the group-velocity dispersion term of nonlinear optical fiber

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Abstract

In this paper, we present a simple numerical technique based on optical frequency domain reflectometry. Nonlinear Schrödinger having anti-cubic equation is suggested for computing a new solitons-solutions propagation. The dispersion is to be calculated from variations in the frequency chirp rate, when frequency-chirped light is utilized as the light source. We utilize an accurate model of the average dispersion, in fibers up to several tens of kilometers. The numerical simulation is obtained by applying variable perturbation theory on the group velocity dispersion term in nonlinear Schrödinger having anti-cubic equation. The proposed model is used with some particular conditions; to be significantly more accurate for links with strong nonlinearities and high dispersion.

1 Introduction

Mathematical models are often utilized to get a fundamental knowledge of optical fiber propagation processes. The nonlinear Schrödinger equation

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(NLSE) is a partial differential equation (PDE) that governs the propagation through an optical fiber. Attenuation, second order dispersion, and Kerr nonlinearity are the three primary impacts of that PDE. The NLSE does not have an analytical response for an arbitrary input pulse when the three effects are taken into account collectively. We cite how the numerical method is very useful to solving such as split step Fourier method SSFM [1, 2]. However, in general, each numerical technique has its own benefits and drawbacks. The method's computing complexity becomes a limiting issue for a large number of steps. Finite elements as a numerical technique is used to ignore limits because optical fiber for this paper has a simple rectangular domain [3]. Analytical models also are typically utilized to get around this constraint. It is particularly desired to have an analytical model, that is simple to manipulate mathematically, since it may be utilized to enhance optic fiber transmission. Mathematical tools are used to create better models, since authors approximate the NLSE several analytical methods [4, 5, 6]. The work [7] contains a thorough review of optical channel models. Depending on the approximation that was used to create it, each model will operate in a specific regime. The group-velocity dispersion (GVD) parameter, the Kerr nonlinear coefficient, and the input power are used by the regimes to categorize models [8].

The schematic representation of some major operational regimes and models found in the literature is shown in 1. The fiber degenerates into an additive Gaussian noise (AGN) channel, in the presence of noise, if both the linear and nonlinear coefficients are zero and no intriguing effects of fiber propagation are seen. When the linear coefficient is zero ($D = 0$) see Fig 1, as shown by region 2 of Fig 1 [16, 17, 9], one of the easiest regimes to account for fiber propagation effects. When the nonlinear coefficients are zero $\left(b_i = 0, i = \overline{1,3}\right)$ see Fig 1, a different straightforward model appears, which is depicted by region 1 of Fig 1. The so-called dispersion-only model provides an accurate solution in this situation [16]. According to the NLSE [16], the phase response of the all-pass filter that represents fiber propagation rises with the square of the frequency. The model is perfect for low power regimes where the dispersion is the primary influence since it takes zero nonlinearities into account.

Region 3 in Fig 1 represents the regular perturbation (RP) theory on the nonlinear coefficient, which is a more appropriate model [10, 18]. As will be shown in the nonlinear term of NLSE, the nonlinearities rely on the signal times the square of the absolute magnitude of the signal. If we compare to the dispersion-only model, RP on the nonlinear coefficient is accurate over a larger range of powers. Due to its greater scope, the RP model can simulate

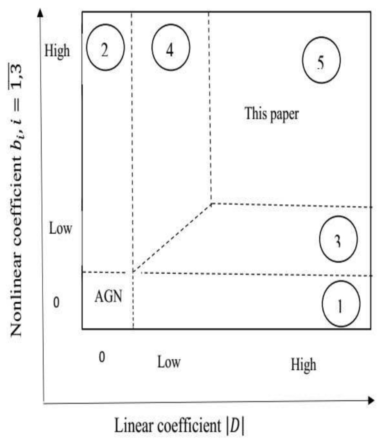

Figure 1: Validity ranges for different models of the NLSE. Each region is identified by a combination of $|D|$ and $b_i, i = \overline{1,3}$ values. These models are derived using approximations based on the magnitude of these two parameters.

a variety of communication systems.
As a model for the weak-dispersion regime symbolized by region 4, authors in [8] suggested a perturbation on the linear coefficient of the NLSE, but in [4, 5, 6] they considered GVD's value as a constant. For more details of other models see [8].
The aforementioned models only cover Fig 1's regions 1, 2, 3, and 4. There are no models about region 5 in the available literature. The latter refers to regimes with high linear and nonlinear coefficients that may be fully captured by perturbation models, including their temporal and spatial components. The absolute value of the linear coefficient is significant in region 5.
Every model that currently exists in the literature is constrained to a certain operating regime. Here, we offer an important model for the high dispersion nonlinear optical fiber channel, with anti-cubic (AC), cubic and quintic nonlinear terms. A careful balance between group velocity dispersion (GVD) and nonlinearities, which promotes the sustainability of the soliton propagation, continues to exist. The one dimensional NLSE's high linear dispersion relation with strong nonlinear AC coefficients has received very little attention despite being a universal model. In this paper, we are interested in high GVD linear coefficient of optical fibers according to a frequency-shifted feedback fiber laser models, where both linear GVD and nonlinear anti-cubic coefficients are large and do not exist in the published literature.

The remainder of this research is organized as follows: In section 2, we combine GVD formula with NLSE having AC to get the solitary solutions propagation. In section 3, we solve the NLSE by Comsol multiphysics then we discuss the obtained results, and their novelty compared with previous models. Section 4 presents the conclusions.

2 Mathematical analysis

The dynamics of soliton propagation through an optical fiber having anti-cubic nonlinearity is written as the following NLSE [11]:

$$i\frac{\partial q(t,x)}{\partial t} + D\frac{\partial^2 q(t,x)}{\partial x^2} + \left(b_1|q(t,x)|^{-4} + b_2|q(t,x)|^2 + b_3|q(t,x)|^4\right) q(t,x) = 0, \tag{2.1}$$

where $q(x,t)$ represents the macroscopic complex-valued wave profile, x and t are the spatial and temporal variables, respectively. Furthermore, D is the coefficient of GVD spatial dispersion. Also, $b_1 \neq 0$ there is AC nonlinear term, whereas b_2 and b_3 are cubic and quintic nonlinearity coefficients, respectively [13]. In case $b_3 = 0$, parabolic low nonlinearity happen [14, 15].
We propose the following:

$$\xi = x - \lambda t + x_0 \ with \ \lambda = -2\tau D, \tag{2.2}$$

when τ is a nonzero real constant and x_0 is the position at t_0.
Now, we define GVD's theoretical law of frequency-shifted feedback fiber laser; the GVD D [3] can be directly determined from a change of beat frequency ∂f_B; it is given by:

$$D = \frac{c\partial f_B}{2\lambda_\gamma^2 L\gamma^2 \Delta t}, \tag{2.3}$$

where γ is the light frequency chirp rate, λ_γ is the light wavelength, L is the optical fiber length and c is the speed of light.
From (2.2) we can find:

$$\partial\xi = 2\tau D\partial t, \tag{2.4}$$

also, we can write:

$$\Delta t = \frac{1}{2\tau D}\partial\xi. \tag{2.5}$$

Substituting (2.5) in (2.3) yields:

$$\partial\xi = \frac{c\tau\partial f_B}{\lambda_\gamma^2 L\gamma^2}. \tag{2.6}$$

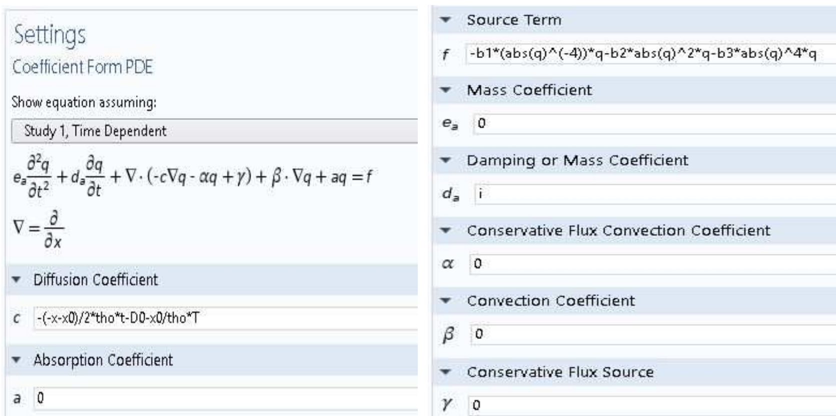

Figure 2: Coupling step: enter the equations to coupling the formula (2.1) with the GVD (2.8).

After integrating (2.6) and using (2.2), we can find the next formula of GVD:

$$D = \frac{-x - x_0}{2\tau t} + \frac{cte\lambda_\gamma{}^2 L\gamma^2}{2\tau t\left(\lambda_\gamma{}^2 L\gamma^2 - \tau c\partial f_B\right)}, \tag{2.7}$$

where $t \neq 0$ and $2\tau\lambda_\gamma^2 c\gamma^2 \neq 2\tau^2\partial f_B$ and cte is the constant integration. At $(t_0 \neq 0, x_0)$ we find $cte = \left(D_0 + \frac{x_0}{\tau t_0}\right)\frac{2\tau t\left(\lambda_\gamma{}^2 L\gamma^2 - \tau c\partial f_B\right)}{\lambda_\gamma{}^2 L\gamma^2}$, where equation (2.7) becomes

$$D = \frac{-x - x_0}{2\tau t} + \frac{x_0}{\tau t_0} + D_0, \tag{2.8}$$

where D_0 is a known initial GVD value.
Boundary and initial conditions: we use [3] to propose and construct the following conditions:

$$q\left(x_0, t\right) = \sin\left(2x_0 + 2\tau D_0 t + x_0\right), \tag{2.9}$$

$$q\left(t\right) = -6sech\left(t\right)^2, x_0 \text{ and } \mathbf{X}, \tag{2.10}$$

with $\mathbf{X}$ is the length of channel and $t \in [0, 1]$.

3 Numerical simulation by Comsol multiphysics

Here, we summarize the important steps to combining (2.1) with (2.8) by using Comsol multiphysics:

Application of periodic boundary condition:

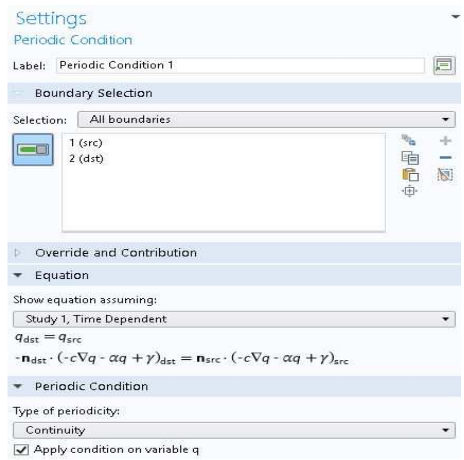

Figure 3: Choice of the boundary conditions step.

Optical periodic solitons-solutions:

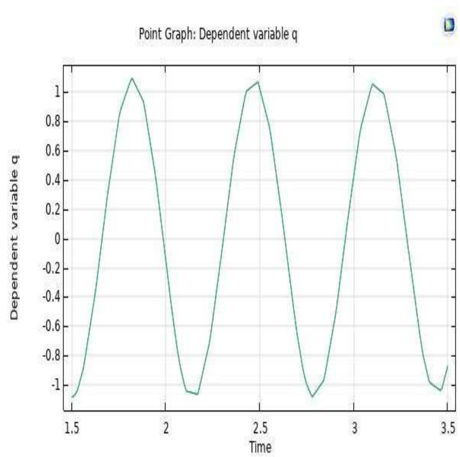

Figure 4: Resulting step: the propagation of solitons-solutions for $x \in [-50, 50]$.

4 Conclusion

The solutions are altered by cubic, quintic and AC terms in a parameterized way. We show that the change of parameters is reasonable according to [3]

see Fig 4. This paper helps to find new style of optical solitons-solutions for NLSE by using second spatiotemporal dispersion non-constant coefficient. The obtaining solitons-solutions respect to various parameters are shown in figure Fig 4. These new solitons with arbitrary parameters may be important to explain optical fiber communications interpretations; we believe that our model can be applied to other fields where the NLSE is applicable. Also, results indicate that simple technique can be extended to other new types of nonlinear partial differential equations.
Cameras may be utilized in water tanks with clear sidewalls to capture the completely spatio-temporal dynamics [12], but our numerical model can be very useful to replace all those materials.

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