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On weakly $\delta(\Lambda, p)$-open functions

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Abstract

Our main purpose is to introduce the concept of weakly $\delta(\Lambda, p)$-open functions. Moreover, we investigate some properties of weakly $\delta(\Lambda, p)$-open functions.

1 Introduction

Open and closed functions are fundamental in the investigation of general topological spaces. Various types of generalizations of open and closed functions have been researched by many mathematicians. Mashhour et al. [9] introduced and studied the notion of preopen functions. Noiri [10] introduced and investigated the concept of semi-open functions. Mashhour et al. [8] studied some characterizations of α-open functions. El-Monsef et al. [1] introduced and investigated the notions of β-open functions. Rose [12] introduced and studied the notions of weakly open functions and almost open functions. Rose and Janković [11] investigated some of the fundamental properties of weakly closed functions. Caldas and Navalagi [6] introduced

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two new classes of functions called weakly preopen functions and weakly preclosed functions as generalization of weak openness and weak closedness due to [12] and [11], respectively. Caldas and Navalagi [5] introduced and discussed the notion of weakly δ-openness as a new generalization of δ-openness and obtained several characterizations of weakly δ-open functions. In [3], the present authors investigated some properties of (Λ, sp)-closed sets and (Λ, sp)-open sets. Boonpok and Viriyapong [4] introduced and studied the notions of (Λ, p)-open sets and (Λ, p)-closed sets. Quite recently, Boonpok and Thongmoon [2] introduced the concepts of $\delta(\Lambda, p)$-closed sets and $\delta(\Lambda, p)$-open sets in topological spaces. In this paper, we introduce the concept of weakly $\delta(\Lambda, p)$-open functions. Moreover, some properties of weakly $\delta(\Lambda, p)$-open functions are investigated.

2 Preliminaries

For a subset A of a topological space (X, τ), $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, represent the closure and the interior of A, respectively. A subset A of a topological space (X, τ) is said to be *preopen* [9] if $A \subseteq \mathrm{Int}(\mathrm{Cl}(A))$. The complement of a preopen set is called *preclosed*. The family of all preopen sets of a topological space (X, τ) is denoted by $PO(X, \tau)$. A subset $\Lambda_p(A)$ [7] is defined as follows: $\Lambda_p(A) = \cap\{U \mid A \subseteq U, U \in PO(X, \tau)\}$. A subset A of a topological space (X, τ) is called a Λ_p-*set* [4] (*pre*-Λ-*set* [7]) if $A = \Lambda_p(A)$. A subset A of a topological space (X, τ) is called (Λ, p)-*closed* [4] if $A = T \cap C$, where T is a Λ_p-set and C is a preclosed set. The complement of a (Λ, p)-closed set is called (Λ, p)-*open*. Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a (Λ, p)-*cluster point* [4] of A if $A \cap U \neq \emptyset$ for every (Λ, p)-open set U of X containing x. The set of all (Λ, p)-cluster points of A is called the (Λ, p)-*closure* [4] of A and is denoted by $A^{(\Lambda,p)}$. The union of all (Λ, p)-open sets of X contained in A is called the (Λ, p)-*interior* [4] of A and is denoted by $A_{(\Lambda,p)}$. The $\theta(\Lambda, p)$-*closure* [4] of A, $A^{\theta(\Lambda,p)}$, is defined as follows: $A^{\theta(\Lambda,p)} = \{x \in X \mid A \cap\ U^{(\Lambda,p)} \neq \emptyset$ for each (Λ, p)-open set U containing $x\}$. A subset A of a topological space (X, τ) is called $\theta(\Lambda, p)$-*closed* [4] if $A = A^{\theta(\Lambda,p)}$. The complement of a $\theta(\Lambda, p)$-closed set is called $\theta(\Lambda, p)$-*open*. A point $x \in X$ is called a $\theta(\Lambda, p)$-*interior point* [13] of A if $x \in U \subseteq U^{(\Lambda,p)} \subseteq A$ for some $U \in \Lambda_pO(X, \tau)$. The set of all $\theta(\Lambda, p)$-interior points of A is called the $\theta(\Lambda, p)$-*interior* [13] of A and is denoted by $A_{\theta(\Lambda,p)}$. A subset A of a topological space (X, τ) is called $r(\Lambda, p)$-*open* [4] (resp. $\alpha(\Lambda, p)$-*open* [14]) if $A = [A^{(\Lambda,p)}]_{(\Lambda,p)}$ (resp $A \subseteq [[A_{(\Lambda,p)}]^{(\Lambda,p)}]_{(\Lambda,p)}$). A point x of X is called a

$\delta(\Lambda, p)$-*cluster point* [2] of A if $A \cap [V^{(\Lambda,p)}]_{(\Lambda,p)} \neq \emptyset$ for every (Λ, p)-open set V of X containing x. The set of all $\delta(\Lambda, p)$-cluster points of A is called the $\delta(\Lambda, p)$-*closure* [2] of A and is denoted by $A^{\delta(\Lambda,p)}$. If $A = A^{\delta(\Lambda,p)}$, then A is said to be $\delta(\Lambda, p)$-*closed* [2]. The complement of a $\delta(\Lambda, p)$-closed set is said to be $\delta(\Lambda, p)$-*open*. The union of all $\delta(\Lambda, p)$-open sets contained in A is called the $\delta(\Lambda, p)$-*interior* [2] of A and is denoted by $A_{\delta(\Lambda,p)}$.

3 Properties of weakly $\delta(\Lambda, p)$-open functions

We begin this section by introducing the concept of weakly $\delta(\Lambda, p)$-open functions.

Definition 3.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be weakly $\delta(\Lambda, p)$-open if $f(U) \subseteq [f(U^{(\Lambda,p)})]_{\delta(\Lambda,p)}$ for every (Λ, p)-open set U of X.*

Theorem 3.2. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is weakly $\delta(\Lambda, p)$-open;

(2) $f(A_{\theta(\Lambda,p)}) \subseteq [f(A)]_{\delta(\Lambda,p)}$ for every subset A of X;

(3) $[f^{-1}(B)]_{\theta(\Lambda,p)} \subseteq f^{-1}(B_{\delta(\Lambda,p)})$ for every subset B of Y;

(4) $f^{-1}(B^{\delta(\Lambda,p)}) \subseteq [f^{-1}(B)]^{\theta(\Lambda,p)}$ for every subset B of Y;

(5) $f(K_{(\Lambda,p)}) \subseteq [f(K)]_{\delta(\Lambda,p)}$ for every (Λ, p)-closed set K of X;

(6) $f([U^{(\Lambda,p)}]_{(\Lambda,p)}) \subseteq [f(U^{(\Lambda,p)})]_{\delta(\Lambda,p)}$ for every (Λ, p)-open set U of X;

(7) $f(U) \subseteq [f(U^{(\Lambda,p)})]_{\delta(\Lambda,p)}$ for every $r(\Lambda, p)$-open set U of X;

(8) $f(U) \subseteq [f(U^{(\Lambda,p)})]_{\delta(\Lambda,p)}$ for every $\alpha(\Lambda, p)$-open set U of X.

Proof. (1) $\Rightarrow$ (2): Let A be any subset of A and $x \in A_{\theta(\Lambda,p)}$. Then, there exists a (Λ, p)-open set U of X such that $x \in U \subseteq U^{(\Lambda,p)} \subseteq A$. Therefore, $f(x) \in f(U) \subseteq f(U^{(\Lambda,p)}) \subseteq f(A)$. Since f is weakly $\delta(\Lambda, p)$-open, $f(U) \subseteq [f(U^{(\Lambda,p)})]_{\delta(\Lambda,p)} \subseteq [f(A)]_{\delta(\Lambda,p)}$. It implies that $f(x) \in [f(A)]_{\delta(\Lambda,p)}$. Then, we have $x \in f^{-1}([f(A)]_{\delta(\Lambda,p)})$. Thus, $A_{\theta(\Lambda,p)} \subseteq f^{-1}([f(A)]_{\delta(\Lambda,p)})$ and hence $f(A_{\theta(\Lambda,p)}) \subseteq [f(A)]_{\delta(\Lambda,p)}$.

(2) $\Rightarrow$ (1): Let U be any (Λ, p)-open set of X. As $U \subseteq [U^{(\Lambda,p)}]_{\theta(\Lambda,p)}$ implies $f(U) \subseteq f([U^{(\Lambda,p)}]_{\theta(\Lambda,p)}) \subseteq [f(U^{(\Lambda,p)})]_{\delta(\Lambda,p)}$. Thus, f is weakly $\delta(\Lambda, p)$-open.

(2) $\Rightarrow$ (3): Let B be any subset of Y. Then by (2), $f([f^{-1}(B)]_{\theta(\Lambda,p)}) \subseteq B_{\delta(\Lambda,p)}$ and hence $[f^{-1}(B)]_{\theta(\Lambda,p)} \subseteq f^{-1}(B_{\delta(\Lambda,p)})$.

(3) $\Rightarrow$ (2): The proof is obvious.

(3) $\Rightarrow$ (4): Let B be any subset of Y. Using (3), we have

$$\begin{aligned} X - [f^{-1}(B)]^{\theta(\Lambda,p)} = [X - f^{-1}(B)]_{\theta(\Lambda,p)} &= [f^{-1}(Y - B)]_{\theta(\Lambda,p)} \\ &\subseteq f^{-1}([Y - B]_{\delta(\Lambda,p)}) \\ &= X - f^{-1}(B^{\delta(\Lambda,p)}) \end{aligned}$$

and hence $f^{-1}(B^{\delta(\Lambda,p)}) \subseteq [f^{-1}(B)]^{\theta(\Lambda,p)}$.

(4) $\Rightarrow$ (3): Let B be any subset of Y. Using (4), we have $X - f^{-1}(B_{\delta(\Lambda,p)}) \subseteq X - [f^{-1}(B)]_{\theta(\Lambda,p)}$. Thus, $[f^{-1}(B)]_{\theta(\Lambda,p)} \subseteq f^{-1}(B_{\delta(\Lambda,p)})$.

(1) $\Rightarrow$ (5) $\Rightarrow$ (6) $\Rightarrow$ (7) $\Rightarrow$ (8) $\Rightarrow$ (1): This is obvious. $\square$

Theorem 3.3. *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective function. Then, the following properties are equivalent:*

(1) f is weakly $\delta(\Lambda, p)$-open;

(2) $[f(U)]^{\delta(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ for every (Λ, p)-open set U of X;

(3) $[f(K_{(\Lambda,p)})]^{\delta(\Lambda,p)} \subseteq f(K)$ for every (Λ, p)-closed set K of X.

Proof. (1) $\Rightarrow$ (3): Let K be any (Λ, p)-closed set of X. Then, we have $f(X - K) = Y - f(K) \subseteq [f([X - K]^{(\Lambda,p)})]_{\delta(\Lambda,p)}$ and hence $Y - f(K) \subseteq Y - [f(K_{(\Lambda,p)})]^{\delta(\Lambda,p)}$. Thus, $[f(K_{(\Lambda,p)})]^{\delta(\Lambda,p)} \subseteq f(K)$.

(3) $\Rightarrow$ (2): Let U be any (Λ, p)-open set of X. Since $U^{(\Lambda,p)}$ is (Λ, p)-closed and $U \subseteq [U^{(\Lambda,p)}]_{(\Lambda,p)}$, by (3) we have $[f(U)]^{\delta(\Lambda,p)} \subseteq [f([U^{(\Lambda,p)}]_{(\Lambda,p)})]^{\delta(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$.

(2) $\Rightarrow$ (3): Similar to (3) $\Rightarrow$ (2).

(3) $\Rightarrow$ (1): The proof is obvious. $\square$

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