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Properties of (Λ, p)-closed functions

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Abstract

In this paper, we introduce the notion of (Λ, p)-closed functions. Moreover, we investigate some properties of (Λ, p)-closed functions.

1 Introduction

Preclosed, semi-closed, α-closed, and β-closed sets play an important role in the search of generalizations of closed functions. By using these sets, several authors introduced and investigated various types of modifications of closed functions. In 1982, Mashhour et al. [6] introduced and studied the notions of preopen functions and preclosed functions. Noiri [8] introduced and investigated the concepts of semi-open functions and semi-closed functions. Mashhour et al. [5] studied some characterizations of α-open functions and α-closed functions. In 1983, El-Monsef et al. [1] introduced and investigated the notions of β-open functions and β-closed functions. In 2006, Noiri and Popa [7] introduced a new class of functions called M-closed functions as functions defined between sets satisfying some conditions and obtained several characterizations of M-closed functions. In [2], the present authors

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investigated some properties of (Λ, sp)-closed sets and (Λ, sp)-open sets. In 2022, Boonpok and Viriyapong [3] introduced and studied the notions of (Λ, p)-open sets and (Λ, p)-closed sets. In this paper, we introduce the notion of (Λ, p)-closed functions. Moreover, we investigate some properties of (Λ, p)-closed functions.

2 Preliminaries

For a subset A of a topological space (X, τ), $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, represent the closure and the interior of A, respectively. A subset A of a topological space (X, τ) is said to be *preopen* [6] if $A \subseteq \mathrm{Int}(\mathrm{Cl}(A))$. The complement of a preopen set is called *preclosed*. The family of all preopen sets of a topological space (X, τ) is denoted by $PO(X, \tau)$. A subset $\Lambda_p(A)$ [4] is defined as follows: $\Lambda_p(A) = \cap\{U \mid A \subseteq U, U \in PO(X, \tau)\}$. A subset A of a topological space (X, τ) is called a Λ_p-*set* [3] (*pre*-Λ-*set* [4]) if $A = \Lambda_p(A)$. A subset A of a topological space (X, τ) is called (Λ, p)-*closed* [3] if $A = T \cap C$, where T is a Λ_p-set and C is a preclosed set. The complement of a (Λ, p)-closed set is called (Λ, p)-*open*. The family of all (Λ, p)-open (resp. (Λ, p)-closed) sets in a topological space (X, τ) is denoted by $\Lambda_p O(X, \tau)$ (resp. $\Lambda_p C(X, \tau)$). Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a (Λ, p)-*cluster point* [3] of A if $A \cap U \neq \emptyset$ for every (Λ, p)-open set U of X containing x. The set of all (Λ, p)-cluster points of A is called the (Λ, p)-*closure* [3] of A and is denoted by $A^{(\Lambda,p)}$. The union of all (Λ, p)-open sets of X contained in A is called the (Λ, p)-*interior* [3] of A and is denoted by $A_{(\Lambda,p)}$.

3 Properties of (Λ, p)-closed functions

We begin this section by introducing the concept of (Λ, p)-closed functions.

Definition 3.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be (Λ, p)-closed if for each (Λ, p)-closed set K of X, $f(K)$ is (Λ, p)-closed in Y.*

Lemma 3.2. *Let A be a subset of a topological space (X, τ). Then, $x \in A^{(\Lambda,p)}$ if and only if $U \cap A \neq \emptyset$ for every $U \in \Lambda_p O(X, \tau)$ containing x.*

Theorem 3.3. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is (Λ, p)-closed;

(2) *for each subset B of Y and each (Λ, p)-open set of X with $f^{-1}(B) \subseteq U$, there exists a (Λ, p)-open set V of Y such that $B \subseteq V$ and $f^{-1}(V) \subseteq U$;*

(3) *for each point $y \in Y$ and each (Λ, p)-open set U of X with $f^{-1}(y) \subseteq U$, there exists a (Λ, p)-open set V of Y containing y such that $f^{-1}(V) \subseteq U$.*

Proof. (1) $\Rightarrow$ (2): Let B be any subset of Y and $U \in \Lambda_p O(X, \tau)$ with $f^{-1}(B) \subseteq U$. Put $V = Y - f(X - U)$. Then, $f(X - U)$ is (Λ, p)-closed and hence V is (Λ, p)-open in Y, $B \subseteq V$ and $f^{-1}(V) \subseteq U$.

(2) $\Rightarrow$ (3): This is obvious.

(3) $\Rightarrow$ (1): Let K be any (Λ, p)-closed set of X and $y \in Y - f(K)$. Since $f^{-1}(y) \subseteq X - K$ and $X - K$ is (Λ, p)-open, there exists a (Λ, p)-open set V with $y \in V$ and $f^{-1}(V) \subseteq X - K$. By Lemma 3.2, $y \in Y - [f(K)]^{(\Lambda,p)}$. Thus, $[f(K)]^{(\Lambda,p)} = f(K)$ and hence $f(K)$ is (Λ, p)-closed. This shows that f is (Λ, p)-closed. □

Theorem 3.4. *A function $f : (X, \tau) \to (Y, \sigma)$ is (Λ, p)-closed if and only if $[f(A)]^{(\Lambda,p)} \subseteq f(A^{(\Lambda,p)})$ for every subset A of X.*

Proof. Suppose that f is (Λ, p)-closed. Let A be any subset of X. Then, $A^{(\Lambda,p)}$ is (Λ, p)-closed. Since f is (Λ, p)-closed, $f(A^{(\Lambda,p)})$ is (Λ, p)-closed. Since $f(A) \subseteq f(A^{(\Lambda,p)})$, $[f(A)]^{(\Lambda,p)} \subseteq [f(A^{(\Lambda,p)})]^{(\Lambda,p)} = f(A^{(\Lambda,p)})$. Therefore, $[f(A)]^{(\Lambda,p)} \subseteq f(A^{(\Lambda,p)})$.

Conversely, let K be any (Λ, p)-closed set of X. Then, by the hypothesis, $[f(K)]^{(\Lambda,p)} \subseteq f(K^{(\Lambda,p)}) = f(K)$. Thus, $[f(K)]^{(\Lambda,p)} = f(K)$ and hence $f(K)$ is (Λ, p)-closed. This shows that f is (Λ, p)-closed. □

Theorem 3.5. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) *f is (Λ, p)-closed.*

(2) *If U is (Λ, p)-open, then the set $V = \{y \in Y \mid f^{-1}(y) \subseteq U\}$ is (Λ, p)-open.*

(3) *If K is (Λ, p)-closed, then the set $H = \{y \in Y \mid f^{-1}(y) \cap K \neq \emptyset\}$ is (Λ, p)-closed.*

Proof. (1) $\Rightarrow$ (2): Let U be any (Λ, p)-open set of X and $y \in V$. By Theorem 3.3, there exists a (Λ, p)-open set W of Y containing y such that $f^{-1}(W) \subseteq U$. Thus, $W \subseteq V$ and $V = V_{(\Lambda,p)}$. Thus, V is (Λ, p)-open.

(2) $\Rightarrow$ (1): Let K be any (Λ, p)-open set of X. Then, $X - K$ is (Λ, p)-open and by (2), $V = \{y \in Y \mid f^{-1}(y) \subseteq X - K\}$. Since $f(K) = Y - V$, it follows that $f(K)$ is (Λ, p)-closed. Thus, f is (Λ, p)-closed.

(2) $\Leftrightarrow$ (3): This is obvious. $\square$

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