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Strongly $\theta(\Lambda, p)$-continuous functions

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Abstract

In this paper, we introduce a new class of functions called strongly $\theta(\Lambda, p)$-continuous functions. Moreover, we investigate several characterizations and some properties concerning strongly $\theta(\Lambda, p)$-continuous functions.

1 Introduction

In 1941, Fomin [3] introduced the notion of θ-continuous functions. In 1980, Noiri [12] introduced the notion of strongly θ-continuous functions. Long et al. [7] studied some properties of strongly θ-continuous functions. In 1998, Jafari and Noiri [6] introduced and studied the notion of strongly θ-semi-continuous functions. Moreover, Jafari and Noiri [5] studied the notion of strongly sober θ-continuous functions. In 2001, Noiri [11] introduced the notion of θ-precontinuous functions. In 2002, Noiri and Popa [10] introduced and investigated the notion of strongly θ-β-continuous functions. Boonpok [2] introduced and studied the notion of M-continuous functions. In 2005, Noiri and Popa [9] defined a new notion of strongly θ-M-continuous functions as

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functions from a set satisfying some minimal conditions into a set satisfying some minimal conditions. Several characterizations and some properties of strongly θ-M-continuous functions were investigated in [9]. Viriyapong and Boonpok [13] studied some properties of (Λ, sp)-continuous functions. Quite recently, Boonpok and Viriyapong [1] introduced and studied the notions of (Λ, p)-open sets and (Λ, p)-closed sets in topological spaces. In this paper, we introduce the notion of strongly $\theta(\Lambda, p)$-continuous functions. Moreover, we investigate several characterizations of strongly $\theta(\Lambda, p)$-continuous functions.

2 Preliminaries

Throughout the present paper, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a topological space (X, τ), $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, represent the closure and the interior of A, respectively. A subset A of a topological space (X, τ) is said to be *preopen* [8] if $A \subseteq \mathrm{Int}(\mathrm{Cl}(A))$. The complement of a preopen set is called *preclosed*. The family of all preopen sets of a topological space (X, τ) is denoted by $PO(X, \tau)$. A subset $\Lambda_p(A)$ [4] is defined as follows: $\Lambda_p(A) = \cap\{U \mid A \subseteq U, U \in PO(X, \tau)\}$. A subset A of a topological space (X, τ) is called a Λ_p-*set* [1] (*pre-*Λ-*set* [4]) if $A = \Lambda_p(A)$. A subset A of a topological space (X, τ) is called (Λ, p)-*closed* [1] if $A = T \cap C$, where T is a Λ_p-set and C is a preclosed set. The complement of a (Λ, p)-closed set is called (Λ, p)-*open*. The family of all (Λ, p)-open (resp. (Λ, p)-closed) sets in a topological space (X, τ) is denoted by $\Lambda_pO(X, \tau)$ (resp. $\Lambda_pC(X, \tau)$). Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a (Λ, p)-*cluster point* [1] of A if $A \cap U \neq \emptyset$ for every (Λ, p)-open set U of X containing x. The set of all (Λ, p)-cluster points of A is called the (Λ, p)-*closure* [1] of A and is denoted by $A^{(\Lambda,p)}$. The union of all (Λ, p)-open sets of X contained in A is called the (Λ, p)-*interior* [1] of A and is denoted by $A_{(\Lambda,p)}$. The $\theta(\Lambda, p)$-*closure* [1] of A, $A^{\theta(\Lambda,p)}$, is defined as follows: $A^{\theta(\Lambda,p)} = \{x \in X \mid A \cap U^{(\Lambda,p)} \neq \emptyset$ for each (Λ, p)-open set U containing $x\}$. A subset A of a topological space (X, τ) is called $\theta(\Lambda, p)$-*closed* [1] if $A = A^{\theta(\Lambda,p)}$. The complement of a $\theta(\Lambda, p)$-closed set is said to be $\theta(\Lambda, p)$-*open*. A point $x \in X$ is called a $\theta(\Lambda, p)$-*interior point* of A if $x \in U \subseteq U^{(\Lambda,p)} \subseteq A$ for some $U \in \Lambda_pO(X, \tau)$. The set of all $\theta(\Lambda, p)$-interior points of A is called the $\theta(\Lambda, p)$-*interior* of A and is denoted by $A_{\theta(\Lambda,p)}$.

3 Strongly $\theta(\Lambda, p)$-continuous functions

In this section, we introduce the notion of strongly $\theta(\Lambda, p)$-continuous functions. Moreover, some characterizations of strongly $\theta(\Lambda, p)$-continuous functions are investigated.

Definition 3.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be strongly $\theta(\Lambda, p)$-continuous at $x \in X$ if for each (Λ, p)-open set V of Y containing $f(x)$, there exists a (Λ, p)-open set U of X containing x such that $f(U^{(\Lambda,p)}) \subseteq V$. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be strongly $\theta(\Lambda, p)$-continuous if f has this property at each point $x \in X$.*

Theorem 3.2. *A function $f : (X, \tau) \to (Y, \sigma)$ is strongly $\theta(\Lambda, p)$-continuous at $x \in X$ if and only if for each (Λ, p)-open set V of Y containing $f(x)$, $x \in [f^{-1}(V)]_{\theta(\Lambda,p)}$.*

Proof. Let $x \in X$ and f be strongly $\theta(\Lambda, p)$-continuous at x. Let V be any (Λ, p)-open set V of Y containing $f(x)$. Then, there exists a (Λ, p)-open set U of X containing x such that $f(U^{(\Lambda,p)}) \subseteq V$. Thus, $x \in U \subseteq U^{(\Lambda,p)} \subseteq f^{-1}(V)$ and hence $x \in [f^{-1}(V)]_{\theta(\Lambda,p)}$.

Conversely, let V be any (Λ, p)-open set V of Y containing $f(x)$. Then, by the hypothesis, $x \in [f^{-1}(V)]_{\theta(\Lambda,p)}$. There exists a (Λ, p)-open set U of X such that $x \in U \subseteq U^{(\Lambda,p)} \subseteq f^{-1}(V)$; hence $f(U^{(\Lambda,p)}) \subseteq V$. This shows that f is strongly $\theta(\Lambda, p)$-continuous at x. □

Lemma 3.3. *For subsets A and B of a topological space (X, τ), the following properties hold:*

(1) $X - A^{\theta(\Lambda,p)} = [X - A]_{\theta(\Lambda,p)}$ *and* $X - A_{\theta(\Lambda,p)} = [X - A]^{\theta(\Lambda,p)}$.

(2) A *is* $\theta(\Lambda, p)$*-open if and only if* $A = A_{\theta(\Lambda,p)}$.

(3) $A \subseteq A^{(\Lambda,p)} \subseteq A^{\theta(\Lambda,p)}$ *and* $A_{\theta(\Lambda,p)} \subseteq A_{(\Lambda,p)} \subseteq A$.

(4) If $A \subseteq B$, *then* $A^{\theta(\Lambda,p)} \subseteq B^{\theta(\Lambda,p)}$ *and* $A_{\theta(\Lambda,p)} \subseteq B_{\theta(\Lambda,p)}$.

(5) If A *is* (Λ, p)*-open, then* $A^{(\Lambda,p)} = A^{\theta(\Lambda,p)}$.

Theorem 3.4. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f *is strongly* $\theta(\Lambda, p)$*-continuous;*

(2) $f^{-1}(V)$ *is* $\theta(\Lambda, p)$*-open in* X *for every* (Λ, p)*-open set* V *of* Y*;*

(3) $f^{-1}(K)$ *is* $\theta(\Lambda,p)$*-closed in* X *for every* (Λ,p)*-closed set* K *of* Y*;*

(4) $f(A^{\theta(\Lambda,p)}) \subseteq [f(A)]^{(\Lambda,p)}$ *for every subset* A *of* X*;*

(5) $[f^{-1}(B)]^{\theta(\Lambda,p)} \subseteq f^{-1}(B^{(\Lambda,p)})$ *for every subset* B *of* Y*.*

Proof. (1) $\Rightarrow$ (2): Let V be any (Λ,p)-open set of Y and $x \in f^{-1}(V)$. Then, $f(x) \in V$. Since f is strongly $\theta(\Lambda,p)$-continuous at x, by Theorem 3.2, $x \in [f^{-1}(V)]_{\theta(\Lambda,p)}$. Thus, $f^{-1}(V) \subseteq [f^{-1}(V)]_{\theta(\Lambda,p)}$ and hence $f^{-1}(V) = [f^{-1}(V)]_{\theta(\Lambda,p)}$. By Lemma 3.3, $f^{-1}(V)$ is $\theta(\Lambda,p)$-open in X.

(2) $\Rightarrow$ (3): Let K be any (Λ,p)-closed set of Y. Then, $Y - K$ is (Λ,p)-open. By Lemma 3.3, $X - f^{-1}(K) = f^{-1}(Y - K) = [f^{-1}(Y - K)]_{\theta(\Lambda,p)} = X - [f^{-1}(K)]^{\theta(\Lambda,p)}$. Thus, $f^{-1}(K) = [f^{-1}(K)]^{\theta(\Lambda,p)}$ and hence $f^{-1}(K)$ is $\theta(\Lambda,p)$-closed in X.

(3) $\Rightarrow$ (1): Let $x \in X$ and V be any (Λ,p)-open set of Y containing $f(x)$. By (3), $f^{-1}(Y - V)$ is $\theta(\Lambda,p)$-closed and $f^{-1}(V)$ is $\theta(\Lambda,p)$-open. By Lemma 3.3, $f^{-1}(V) = [f^{-1}(V)]_{\theta(\Lambda,p)}$. Since $x \in [f^{-1}(V)]_{\theta(\Lambda,p)}$, there exists a (Λ,p)-open set U of X such that $x \in U \subseteq U^{(\Lambda,p)} \subseteq f^{-1}(V)$. Thus, $f(U^{(\Lambda,p)}) \subseteq V$. This shows that f is strongly $\theta(\Lambda,p)$-continuous.

(1) $\Rightarrow$ (4): Let A be any subset of X. Suppose that $x \in A^{\theta(\Lambda,p)}$. Let V be any (Λ,p)-open set of Y containing $f(x)$. Since f is strongly $\theta(\Lambda,p)$-continuous, there exists a (Λ,p)-open set U of X containing x such that $f(U^{(\Lambda,p)}) \subseteq V$. Since $x \in A^{\theta(\Lambda,p)}$, we have $U^{(\Lambda,p)} \cap A \neq \emptyset$. It follows that $\emptyset \neq f(U^{(\Lambda,p)}) \cap f(A) \subseteq V \cap f(A)$. Thus, $f(x) \in [f(A)]^{(\Lambda,p)}$.

(4) $\Rightarrow$ (5): Let B be any subset of Y. Then, we have $f([f^{-1}(B)]^{\theta(\Lambda,p)}) \subseteq [f(f^{-1}(B)]^{(\Lambda,p)} \subseteq B^{(\Lambda,p)}$ and hence $[f^{-1}(B)]^{\theta(\Lambda,p)} \subseteq f^{-1}(B^{(\Lambda,p)})$.

(5) $\Rightarrow$ (1): Let $x \in X$ and V be any (Λ,p)-open set of Y containing $f(x)$. Since $V \cap (Y - V) = \emptyset$, by Lemma 3.3 $f(x) \notin [Y - V]^{(\Lambda,p)}$ and hence $x \notin f^{-1}([Y - V]^{(\Lambda,p)})$. By (5), $x \notin [Y - V]^{\theta(\Lambda,p)} = X - [f^{-1}(V)]_{\theta(\Lambda,p)}$ and hence $x \in [f^{-1}(V)]_{\theta(\Lambda,p)}$. There exists a (Λ,p)-open set U of X containing x such that $U^{(\Lambda,p)} \subseteq f^{-1}(V)$; hence $f(U^{(\Lambda,p)}) \subseteq V$. This shows that f is strongly $\theta(\Lambda,p)$-continuous. □

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