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$\theta(\Lambda, p)$-continuity for functions

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Abstract

This paper is concerned with the notion of $\theta(\Lambda, p)$-continuous functions. Moreover, some characterizations and several properties concerning $\theta(\Lambda, p)$-continuous functions are investigated.

1 Introduction

The concept of θ-continuous functions was first introduced by Fomin [6]. Noiri [13] studied some properties of θ-continuous functions. Arya and Bhamini [1] introduced the notion of θ-semi-continuous functions. Noiri [12] investigated several characterizations of θ-semi-continuous functions. Moreover, Jafari and Noiri [8] obtained some properties of θ-semi-continuous functions. Di Maio and Noiri [4] introduced the concept of quasi-irresolute functions. It is shown in [3] that a function is quasi-irresolute if and only if it is θ-irresolute in the sense of Dube et al. [5]. Noiri [11] introduced and investigated the notion of θ-preirresolute functions. The notion of weakly β-irresolute functions has been defined and studied in [10]. These four classes of functions have properties similar to the class of θ-continuous functions.

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Viriyapong and Boonpok [15] investigated some characterizations of (Λ, sp)-continuous functions. In [2], the present authors introduced and studied the notions of (Λ, p)-open sets and (Λ, p)-closed sets. In this paper, we introduce the concept of $\theta(\Lambda, p)$-continuous functions. Moreover, some characterizations of $\theta(\Lambda, p)$-continuous functions are investigated.

2 Preliminaries

Throughout the present paper, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a topological space (X, τ), $\text{Cl}(A)$ and $\text{Int}(A)$, represent the closure and the interior of A, respectively. A subset A of a topological space (X, τ) is said to be *preopen* [9] if $A \subseteq \text{Int}(\text{Cl}(A))$. The complement of a preopen set is called *preclosed*. The family of all preopen sets of a topological space (X, τ) is denoted by $PO(X, \tau)$. A subset $\Lambda_p(A)$ [7] is defined as follows: $\Lambda_p(A) = \cap\{U \mid A \subseteq U, U \in PO(X, \tau)\}$. A subset A of a topological space (X, τ) is called a Λ_p*-set* [2] (*pre-*Λ*-set* [7]) if $A = \Lambda_p(A)$. A subset A of a topological space (X, τ) is called (Λ, p)*-closed* [2] if $A = T \cap C$, where T is a Λ_p-set and C is a preclosed set. The complement of a (Λ, p)-closed set is called (Λ, p)*-open*. The family of all (Λ, p)-open (resp. (Λ, p)-closed) sets in a topological space (X, τ) is denoted by $\Lambda_p O(X, \tau)$ (resp. $\Lambda_p C(X, \tau)$). Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a (Λ, p)*-cluster point* [2] of A if $A \cap U \neq \emptyset$ for every (Λ, p)-open set U of X containing x. The set of all (Λ, p)-cluster points of A is called the (Λ, p)*-closure* [2] of A and is denoted by $A^{(\Lambda,p)}$. The union of all (Λ, p)-open sets of X contained in A is called the (Λ, p)*-interior* [2] of A and is denoted by $A_{(\Lambda,p)}$. The $\theta(\Lambda, p)$*-closure* [2] of A, $A^{\theta(\Lambda,p)}$, is defined as follows: $A^{\theta(\Lambda,p)} = \{x \in X \mid A \cap U^{(\Lambda,p)} \neq \emptyset$ for each (Λ, p)-open set U containing $x\}$. A subset A of a topological space (X, τ) is called $\theta(\Lambda, p)$*-closed* [2] if $A = A^{\theta(\Lambda,p)}$. The complement of a $\theta(\Lambda, p)$-closed set is said to be $\theta(\Lambda, p)$*-open*. A point $x \in X$ is called a $\theta(\Lambda, p)$*-interior point* [14] of A if $x \in U \subseteq U^{(\Lambda,p)} \subseteq A$ for some $U \in \Lambda_p O(X, \tau)$. The set of all $\theta(\Lambda, p)$-interior points of A is called the $\theta(\Lambda, p)$*-interior* [14] of A and is denoted by $A_{\theta(\Lambda,p)}$.

3 $\theta(\Lambda, p)$-continuous functions

We begin this section by introducing the concept of $\theta(\Lambda, p)$-continuous functions.

Definition 3.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be $\theta(\Lambda, p)$-continuous at $x \in X$ if for each (Λ, p)-open set V of Y containing $f(x)$, there exists a (Λ, p)-open set U of X containing x such that $f(U^{(\Lambda,p)}) \subseteq V^{(\Lambda,p)}$. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be $\theta(\Lambda, p)$-continuous if f has this property at each point $x \in X$.*

Theorem 3.2. *A function $f : (X, \tau) \to (Y, \sigma)$ is $\theta(\Lambda, p)$-continuous at $x \in X$ if and only if $x \in [f^{-1}(V^{(\Lambda,p)})]_{\theta(\Lambda,p)}$ for each (Λ, p)-open set V of Y containing $f(x)$.*

Proof. Let $x \in X$ and f be $\theta(\Lambda, p)$-continuous at x. Let V be any (Λ, p)-open set V of Y containing $f(x)$. Then, there exists a (Λ, p)-open set U of X containing x such that $f(U^{(\Lambda,p)}) \subseteq V^{(\Lambda,p)}$. Thus, $x \in U \subseteq U^{(\Lambda,p)} \subseteq f^{-1}(V^{(\Lambda,p)})$ and hence $x \in [f^{-1}(V^{(\Lambda,p)})]_{\theta(\Lambda,p)}$.

Conversely, let V be any (Λ, p)-open set V of Y containing $f(x)$. Then, by the hypothesis, $x \in [f^{-1}(V^{(\Lambda,p)})]_{\theta(\Lambda,p)}$. There exists a (Λ, p)-open set U of X such that $x \in U \subseteq U^{(\Lambda,p)} \subseteq f^{-1}(V^{(\Lambda,p)})$; hence $f(U^{(\Lambda,p)}) \subseteq V^{(\Lambda,p)}$. This shows that f is $\theta(\Lambda, p)$-continuous at x. □

Theorem 3.3. *A function $f : (X, \tau) \to (Y, \sigma)$ is $\theta(\Lambda, p)$-continuous if and only if $f^{-1}(V) \subseteq [f^{-1}(V^{(\Lambda,p)})]_{\theta(\Lambda,p)}$ for each (Λ, p)-open set V of Y.*

Proof. Let V be any (Λ, p)-open set V of Y and $x \in f^{-1}(V)$. Then, $f(x) \in V$. Since f is $\theta(\Lambda, p)$-continuous at x, by Theorem 3.2, $x \in [f^{-1}(V^{(\Lambda,p)})]_{\theta(\Lambda,p)}$ and hence $f^{-1}(V) \subseteq [f^{-1}(V^{(\Lambda,p)})]_{\theta(\Lambda,p)}$.

Conversely, let $x \in X$ and V be any (Λ, p)-open set V of Y containing $f(x)$. Then, $x \in f^{-1}(V) \subseteq [f^{-1}(V^{(\Lambda,p)})]_{\theta(\Lambda,p)}$ and hence $x \in [f^{-1}(V^{(\Lambda,p)})]_{\theta(\Lambda,p)}$. By Theorem 3.2, f is $\theta(\Lambda, p)$-continuous. □

Theorem 3.4. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is $\theta(\Lambda, p)$-continuous;

(2) $f(A^{\theta(\Lambda,p)}) \subseteq [f(A)]^{\theta(\Lambda,p)}$ for every subset A of X;

(3) $[f^{-1}(B)]^{\theta(\Lambda,p)} \subseteq f^{-1}(B^{\theta(\Lambda,p)})$ for every subset B of Y.

Proof. (1) ⇒ (2): Let A be any subset of X. Suppose that $x \in A^{\theta(\Lambda,p)}$. Let V be any (Λ, p)-open set of Y containing $f(x)$. Since f is $\theta(\Lambda, p)$-continuous, there exists a (Λ, p)-open set U of X containing x such that $f(U^{(\Lambda,p)}) \subseteq V^{(\Lambda,p)}$. Since $x \in A^{\theta(\Lambda,p)}$, $U^{(\Lambda,p)} \cap A \neq \emptyset$. It follows that $\emptyset \neq f(U^{(\Lambda,p)}) \cap f(A) \subseteq V^{(\Lambda,p)} \cap f(A)$. Thus, $f(x) \in [f(A)]^{\theta(\Lambda,p)}$.

(2) $\Rightarrow$ (3): Let B be any subset of Y. Then, we have $f([f^{-1}(B)]^{\theta(\Lambda,p)}) \subseteq [f(f^{-1}(B)]^{\theta(\Lambda,p)} \subseteq B^{\theta(\Lambda,p)}$ and hence $[f^{-1}(B)]^{\theta(\Lambda,p)} \subseteq f^{-1}(B^{\theta(\Lambda,p)})$.

(3) $\Rightarrow$ (1): Let $x \in X$ and V be any (Λ,p)-open set of Y containing $f(x)$. Since $V^{(\Lambda,p)} \cap (Y - V^{(\Lambda,p)}) = \emptyset$, clearly $f(x) \notin [Y - V^{(\Lambda,p)}]^{\theta(\Lambda,p)}$ and hence $x \notin f^{-1}([Y - V^{(\Lambda,p)}]^{\theta(\Lambda,p)})$. By (3), $x \notin [f^{-1}(Y - V^{(\Lambda,p)})]^{\theta(\Lambda,p)}$. There exists a (Λ,p)-open set U of X containing x such that $U^{(\Lambda,p)} \cap f^{-1}(Y - V^{(\Lambda,p)}) = \emptyset$; hence $f(U^{(\Lambda,p)}) \cap (Y - V^{(\Lambda,p)}) = \emptyset$. Thus, $f(U^{(\Lambda,p)}) \subseteq V^{(\Lambda,p)}$. This shows that f is $\theta(\Lambda,p)$-continuous. □

Let A be a subset of a topological space (X,τ). The *$\theta(\Lambda,p)$-frontier* of A, $\theta(\Lambda,p)Fr(A)$, is defined by $\theta(\Lambda,p)Fr(A) = A^{\theta(\Lambda,p)} \cap [X - A]^{\theta(\Lambda,p)}$.

Theorem 3.5. *The set of all points $x \in X$ at which a function $f : (X,\tau) \to (Y,\sigma)$ is not $\theta(\Lambda,p)$-continuous is identical with the union of the $\theta(\Lambda,p)$-frontier of the inverse images of the (Λ,p)-closures of (Λ,p)-open sets containing $f(x)$.*

Proof. Suppose that f is not $\theta(\Lambda,p)$-continuous at $x \in X$. There exists a (Λ,p)-open set V of Y containing $f(x)$ such that $f(U^{(\Lambda,p)})$ is not contained in $V^{(\Lambda,p)}$ for every (Λ,p)-open set U of X containing x. Then, we have $U^{(\Lambda,p)} \cap (X - f^{-1}(V^{(\Lambda,p)})) \neq \emptyset$ for every (Λ,p)-open set U containing x and hence $x \in [X - f^{-1}(V^{(\Lambda,p)})]^{\theta(\Lambda,p)}$. On the other hand, we have $x \in f^{-1}(V) \subseteq [f^{-1}(V^{(\Lambda,p)})]^{\theta(\Lambda,p)}$ and hence $x \in \theta(\Lambda,p)Fr(f^{-1}(V^{(\Lambda,p)}))$.

Conversely, suppose that f is $\theta(\Lambda,p)$-continuous at $x \in X$. Let V be any (Λ,p)-open set of Y containing $f(x)$. By Theorem 3.3, $x \in f^{-1}(V) \subseteq [f^{-1}(V^{(\Lambda,p)})]_{\theta(\Lambda,p)}$. Thus, $x \notin \theta(\Lambda,p)Fr(f^{-1}(V^{(\Lambda,p)}))$ for each (Λ,p)-open set V containing $f(x)$. This complete the proof. □

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