\theta(\Lambda, p)\text{-continuity for functions}

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(Received July 8, 2023, Accepted November 2, 2023,
Published November 10, 2023)

Abstract

This paper is concerned with the notion of \theta(\Lambda, p)-continuous functions. Moreover, some characterizations and several properties concerning \theta(\Lambda, p)-continuous functions are investigated.

1 Introduction

The concept of \theta-continuous functions was first introduced by Fomin [6]. Noiri [13] studied some properties of \theta-continuous functions. Arya and Bhamini [1] introduced the notion of \theta-semi-continuous functions. Noiri [12] investigated several characterizations of \theta-semi-continuous functions. Moreover, Jafari and Noiri [8] obtained some properties of \theta-semi-continuous functions. Di Maio and Noiri [4] introduced the concept of quasi-irresolute functions. It is shown in [3] that a function is quasi-irresolute if and only if it is \theta-irresolute in the sense of Dube et al. [5]. Noiri [11] introduced and investigated the notion of \theta-preirresolute functions. The notion of weakly \beta-irresolute functions has been defined and studied in [10]. These four classes of functions have properties similar to the class of \theta-continuous functions.

Key words and phrases: \(\Lambda, p\)-open set, \theta(\Lambda, p)-continuous function.
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AMS (MOS) Subject Classifications: 54A05, 54C08.
ISSN 1814-0432, 2024, http://ijmcs.future-in-tech.net
Viriyapong and Boonpok [15] investigated some characterizations of \((\Lambda, sp)\)-continuous functions. In [2], the present authors introduced and studied the notions of \((\Lambda, p)\)-open sets and \((\Lambda, p)\)-closed sets. In this paper, we introduce the concept of \(\theta(\Lambda, p)\)-continuous functions. Moreover, some characterizations of \(\theta(\Lambda, p)\)-continuous functions are investigated.

2 Preliminaries

Throughout the present paper, spaces \((X, \tau)\) and \((Y, \sigma)\) (or simply \(X\) and \(Y\)) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset \(A\) of a topological space \((X, \tau)\), \(\text{Cl}(A)\) and \(\text{Int}(A)\), represent the closure and the interior of \(A\), respectively. A subset \(A\) of a topological space \((X, \tau)\) is said to be preopen [9] if \(A \subseteq \text{Int}(\text{Cl}(A))\). The complement of a preopen set is called preclosed. The family of all preopen sets of a topological space \((X, \tau)\) is denoted by \(\text{PO}(X, \tau)\). A subset \(\Lambda_p(A)\) [7] is defined as follows: \(\Lambda_p(A) = \cap\{U \mid A \subseteq U, U \in \text{PO}(X, \tau)\}\). A subset \(A\) of a topological space \((X, \tau)\) is called a \(\Lambda_p\)-set [2] (pre-\(\Lambda\)-set [7]) if \(A = \Lambda_p(A)\). A subset \(A\) of a topological space \((X, \tau)\) is called \((\Lambda, p)\)-closed [2] if \(A = T \cap \text{C}(X, \tau)\), where \(T\) is a \(\Lambda_p\)-set and \(C\) is a preclosed set. The complement of a \((\Lambda, p)\)-closed set is called \((\Lambda, p)\)-open. The family of all \((\Lambda, p)\)-open (resp. \((\Lambda, p)\)-closed) sets in a topological space \((X, \tau)\) is denoted by \(\Lambda_pO(X, \tau)\) (resp. \(\Lambda_pC(X, \tau)\)).

Let \(A\) be a subset of a topological space \((X, \tau)\). A point \(x \in X\) is called a \((\Lambda, p)\)-cluster point [2] of \(A\) if \(A \cap U \neq \emptyset\) for every \((\Lambda, p)\)-open set \(U\) containing \(x\). The set of all \((\Lambda, p)\)-cluster points of \(A\) is called the \((\Lambda, p)\)-closure [2] of \(A\) and is denoted by \(A^{(\Lambda, p)}\). The union of all \((\Lambda, p)\)-open sets of \(X\) contained in \(A\) is called the \((\Lambda, p)\)-interior [2] of \(A\) and is denoted by \(A_{(\Lambda, p)}\). The \(\theta(\Lambda, p)\)-closure [2] of \(A\), \(A^{\theta(\Lambda, p)}\), is defined as follows: \(A^{\theta(\Lambda, p)} = \{x \in X \mid A \cap U^{(\Lambda, p)} \neq \emptyset\ \text{for each} \ (\Lambda, p)\)-open set \(U\ \text{containing} \ x\}\). A subset \(A\) of a topological space \((X, \tau)\) is called \(\theta(\Lambda, p)\)-closed [2] if \(A = A^{\theta(\Lambda, p)}\). The complement of a \(\theta(\Lambda, p)\)-closed set is said to be \(\theta(\Lambda, p)\)-open. A point \(x \in X\) is called a \(\theta(\Lambda, p)\)-interior point [14] of \(A\) if \(x \in U \subseteq U^{(\Lambda, p)} \subseteq A\) for some \(U \in \Lambda_pO(X, \tau)\). The set of all \(\theta(\Lambda, p)\)-interior points of \(A\) is called the \(\theta(\Lambda, p)\)-interior [14] of \(A\) and is denoted by \(A_{\theta(\Lambda, p)}\).

3 \(\theta(\Lambda, p)\)-continuous functions

We begin this section by introducing the concept of \(\theta(\Lambda, p)\)-continuous functions.
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**Definition 3.1.** A function \(f : (X, \tau) \to (Y, \sigma)\) is said to be \(\theta(\Lambda, p)\)-continuous at \(x \in X\) if for each \((\Lambda, p)\)-open set \(V\) of \(Y\) containing \(f(x)\), there exists a \((\Lambda, p)\)-open set \(U\) of \(X\) containing \(x\) such that \(f(U^{(\Lambda, p)}) \subseteq V^{(\Lambda, p)}\). A function \(f : (X, \tau) \to (Y, \sigma)\) is said to be \(\theta(\Lambda, p)\)-continuous if \(f\) has this property at each point \(x \in X\).

**Theorem 3.2.** A function \(f : (X, \tau) \to (Y, \sigma)\) is \(\theta(\Lambda, p)\)-continuous at \(x \in X\) if and only if \(x \in [f^{-1}(V^{(\Lambda, p)})]_{\theta(\Lambda, p)}\) for each \((\Lambda, p)\)-open set \(V\) of \(Y\) containing \(f(x)\).

**Proof.** Let \(x \in X\) and \(f\) be \(\theta(\Lambda, p)\)-continuous at \(x\). Let \(V\) be any \((\Lambda, p)\)-open set of \(Y\) containing \(f(x)\). Then, there exists a \((\Lambda, p)\)-open set \(U\) of \(X\) containing \(x\) such that \(f(U^{(\Lambda, p)}) \subseteq V^{(\Lambda, p)}\). Thus, \(x \in U \subseteq U^{(\Lambda, p)} \subseteq f^{-1}(V^{(\Lambda, p)})\) and hence \(x \in [f^{-1}(V^{(\Lambda, p)})]_{\theta(\Lambda, p)}\).

Conversely, let \(V\) be any \((\Lambda, p)\)-open set of \(Y\) containing \(f(x)\). Then, by the hypothesis, \(x \in [f^{-1}(V^{(\Lambda, p)})]_{\theta(\Lambda, p)}\). There exists a \((\Lambda, p)\)-open set \(U\) of \(X\) such that \(x \in U \subseteq U^{(\Lambda, p)} \subseteq f^{-1}(V^{(\Lambda, p)})\); hence \(f(U^{(\Lambda, p)}) \subseteq V^{(\Lambda, p)}\). This shows that \(f\) is \(\theta(\Lambda, p)\)-continuous at \(x\).

**Theorem 3.3.** A function \(f : (X, \tau) \to (Y, \sigma)\) is \(\theta(\Lambda, p)\)-continuous if and only if \(f^{-1}(V) \subseteq [f^{-1}(V^{(\Lambda, p)})]_{\theta(\Lambda, p)}\) for each \((\Lambda, p)\)-open set \(V\) of \(Y\).

**Proof.** Let \(V\) be any \((\Lambda, p)\)-open set of \(Y\) and \(x \in f^{-1}(V)\). Then, \(f(x) \in V\). Since \(f\) is \(\theta(\Lambda, p)\)-continuous at \(x\), by Theorem 3.2, \(x \in [f^{-1}(V^{(\Lambda, p)})]_{\theta(\Lambda, p)}\) and hence \(f^{-1}(V) \subseteq [f^{-1}(V^{(\Lambda, p)})]_{\theta(\Lambda, p)}\).

Conversely, let \(x \in X\) and \(V\) be any \((\Lambda, p)\)-open set of \(Y\) containing \(f(x)\). Then, \(x \in f^{-1}(V) \subseteq [f^{-1}(V^{(\Lambda, p)})]_{\theta(\Lambda, p)}\) and hence \(x \in [f^{-1}(V^{(\Lambda, p)})]_{\theta(\Lambda, p)}\). By Theorem 3.2, \(f\) is \(\theta(\Lambda, p)\)-continuous.

**Theorem 3.4.** For a function \(f : (X, \tau) \to (Y, \sigma)\), the following properties are equivalent:

1. \(f\) is \(\theta(\Lambda, p)\)-continuous;

2. \(f(A^{\theta(\Lambda, p)}) \subseteq [f(A)]^{\theta(\Lambda, p)}\) for every subset \(A\) of \(X\);

3. \([f^{-1}(B)]^{\theta(\Lambda, p)} \subseteq f^{-1}(B^{\theta(\Lambda, p)})\) for every subset \(B\) of \(Y\).

**Proof.** (1) \(\Rightarrow\) (2): Let \(A\) be any subset of \(X\). Suppose that \(x \in A^{\theta(\Lambda, p)}\). Let \(V\) be any \((\Lambda, p)\)-open set of \(Y\) containing \(f(x)\). Since \(f\) is \(\theta(\Lambda, p)\)-continuous, there exists a \((\Lambda, p)\)-open set \(U\) of \(X\) containing \(x\) such that \(f(U^{(\Lambda, p)}) \subseteq V^{(\Lambda, p)}\). Since \(x \in A^{\theta(\Lambda, p)}\), \(U^{(\Lambda, p)} \cap A \neq \emptyset\). It follows that \(\emptyset \neq f(U^{(\Lambda, p)}) \cap f(A) \subseteq V^{(\Lambda, p)} \cap f(A)\). Thus, \(f(x) \in [f(A)]^{\theta(\Lambda, p)}\).
(2) $\Rightarrow$ (3): Let $B$ be any subset of $Y$. Then, we have $f([f^{-1}(B)]_{\theta(\Lambda, p)}) \subseteq [f^{-1}(B)]_{\theta(\Lambda, p)} \subseteq B_{\theta(\Lambda, p)}$ and hence $[f^{-1}(B)]_{\theta(\Lambda, p)} \subseteq f^{-1}(B_{\theta(\Lambda, p)})$.

(3) $\Rightarrow$ (1): Let $x \in X$ and $V$ be any $(\Lambda, p)$-open set of $Y$ containing $f(x)$. Since $V_{\theta(\Lambda, p)} \cap (Y - V_{\theta(\Lambda, p)}) = \emptyset$, clearly $f(x) \notin [Y - V_{\theta(\Lambda, p)}]_{\theta(\Lambda, p)}$ and hence $x \notin f^{-1}([Y - V_{\theta(\Lambda, p)}]_{\theta(\Lambda, p)})$. By (3), $x \notin [f^{-1}(Y - V_{\theta(\Lambda, p)})]_{\theta(\Lambda, p)}$. There exists a $(\Lambda, p)$-open set $U$ of $X$ containing $x$ such that $U_{\theta(\Lambda, p)} \cap f^{-1}(Y - V_{\theta(\Lambda, p)}) = \emptyset$; hence $f(U_{\theta(\Lambda, p)}) \cap (Y - V_{\theta(\Lambda, p)}) = \emptyset$. Thus, $f(U_{\theta(\Lambda, p)}) \subseteq V_{\theta(\Lambda, p)}$. This shows that $f$ is $\theta(\Lambda, p)$-continuous. 

Let $A$ be a subset of a topological space $(X, \tau)$. The $\theta(\Lambda, p)$-frontier of $A$, $\theta(\Lambda, p)Fr(A)$, is defined by $\theta(\Lambda, p)Fr(A) = A^{\theta(\Lambda, p)} \cap [X - A]^{\theta(\Lambda, p)}$.

**Theorem 3.5.** The set of all points $x \in X$ at which a function $f : (X, \tau) \to (Y, \sigma)$ is not $\theta(\Lambda, p)$-continuous is identical with the union of the $\theta(\Lambda, p)$-frontier of the inverse images of the $(\Lambda, p)$-closures of $(\Lambda, p)$-open sets containing $f(x)$.

**Proof.** Suppose that $f$ is not $\theta(\Lambda, p)$-continuous at $x \in X$. There exists a $(\Lambda, p)$-open set $V$ of $Y$ containing $f(x)$ such that $f(U_{\theta(\Lambda, p)})$ is not contained in $V_{\theta(\Lambda, p)}$ for every $(\Lambda, p)$-open set $U$ of $X$ containing $x$. Then, we have $U_{\theta(\Lambda, p)} \cap (X - f^{-1}(V_{\theta(\Lambda, p)})) \neq \emptyset$ for every $(\Lambda, p)$-open set $U$ containing $x$ and hence $x \in [X - f^{-1}(V_{\theta(\Lambda, p)})]_{\theta(\Lambda, p)}$. On the other hand, we have $x \in f^{-1}(V) \subseteq [f^{-1}(V_{\theta(\Lambda, p)})]_{\theta(\Lambda, p)}$ and hence $x \in \theta(\Lambda, p)Fr(f^{-1}(V_{\theta(\Lambda, p)}))$.

Conversely, suppose that $f$ is $\theta(\Lambda, p)$-continuous at $x \in X$. Let $V$ be any $(\Lambda, p)$-open set of $Y$ containing $f(x)$. By Theorem 3.3, $x \in f^{-1}(V) \subseteq [f^{-1}(V_{\theta(\Lambda, p)})]_{\theta(\Lambda, p)}$. Thus, $x \notin \theta(\Lambda, p)Fr(f^{-1}(V_{\theta(\Lambda, p)}))$ for each $(\Lambda, p)$-open set $V$ containing $f(x)$. This completes the proof. \(\square\)

**Acknowledgment.** This research project was financially supported by Mahasarakham University.

**References**


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