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Some properties of weakly $b(\Lambda, p)$-open functions

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Abstract

In this article, we introduce the concept of weakly $b(\Lambda, p)$-open functions. Moreover, we investigate some properties of weakly $b(\Lambda, p)$-open functions.

1 Introduction

In 1983, Rose [6] introduced and studied the notions of weakly open functions and almost open functions. In 1987, Rose and Janković [5] investigated some of the fundamental properties of weakly closed functions. In 1996, Andrijević [1] introduced a new class of generalized open sets called b-open sets in a topological space. In 2009, Noiri [3] introduced and investigated the class of weakly b-open functions as a new generalization of weakly open functions. Moreover, Noiri et al. [4] introduced and studied two new classes of functions called weakly b-θ-open functions and weakly b-θ-open functions by utilizing the notions of b-θ-open sets and the b-θ-closure operator. Weak b-θ-openness (resp. b-θ-closedness) is a generalization of both θ-preopenness and weak semi-θ-openness (resp. θ-preclosedness and weak semi-θ-closedness).

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Recently, Boonpok and Viriyapong [2] introduced and studied the concepts of (Λ, p)-closed sets and (Λ, p)-open sets in topological spaces. In this article, we introduce the notion of weakly $b(\Lambda, p)$-open functions. Furthermore, several properties of weakly $b(\Lambda, p)$-open functions are discussed.

2 Preliminaries

A subset A of a topological space (X, τ) is called (Λ, p)-*closed* [2] if $A = T \cap C$, where T is a Λ_p-set and C is a preclosed set. The complement of a (Λ, p)-closed set is called (Λ, p)-*open*. The family of all (Λ, p)-open (resp. (Λ, p)-closed) sets in a topological space (X, τ) is denoted by $\Lambda_p O(X, \tau)$ (resp. $\Lambda_p C(X, \tau)$). Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a (Λ, p)-*cluster point* [2] of A if $A \cap U \neq \emptyset$ for every (Λ, p)-open set U of X containing x. The set of all (Λ, p)-cluster points of A is called the (Λ, p)-*closure* [2] of A and is denoted by $A^{(\Lambda,p)}$. The union of all (Λ, p)-open sets of X contained in A is called the (Λ, p)-*interior* [2] of A and is denoted by $A_{(\Lambda,p)}$. The $\theta(\Lambda, p)$-*closure* [2] of A, $A^{\theta(\Lambda,p)}$, is defined as follows: $A^{\theta(\Lambda,p)} = \{x \in X \mid A \cap\ U^{(\Lambda,p)} \neq \emptyset$ for each (Λ, p)-open set U containing $x\}$. A subset A of a topological space (X, τ) is called $\theta(\Lambda, p)$-*closed* [2] if $A = A^{\theta(\Lambda,p)}$. The complement of a $\theta(\Lambda, p)$-closed set is said to be $\theta(\Lambda, p)$-*open*. A point $x \in X$ is called a $\theta(\Lambda, p)$-*interior point* [7] of A if $x \in U \subseteq U^{(\Lambda,p)} \subseteq A$ for some $U \in \Lambda_p O(X, \tau)$. The set of all $\theta(\Lambda, p)$-interior points of A is called the $\theta(\Lambda, p)$-*interior* [7] of A and is denoted by $A_{\theta(\Lambda,p)}$. A subset A of a topological space (X, τ) is said to be $b(\Lambda, p)$-*open* (resp. $p(\Lambda, p)$-*open* [2] $\alpha(\Lambda, p)$-*open*, [8]) if $A \subseteq [A^{(\Lambda,p)}]_{(\Lambda,p)} \cup [A_{(\Lambda,p)}]^{(\Lambda,p)}$ (resp. $A \subseteq [A^{(\Lambda,p)}]_{(\Lambda,p)}$, $A \subseteq [[A_{(\Lambda,p)}]^{(\Lambda,p)}]_{(\Lambda,p)}$). The union of all $b(\Lambda, p)$-open sets of X contained in A is called the $b(\Lambda, p)$-*interior* of A and is denoted by $A_{b(\Lambda,p)}$. The complement of a $b(\Lambda, p)$-open (resp. $p(\Lambda, p)$-open, $\alpha(\Lambda, p)$-open) set is called $b(\Lambda, p)$-*closed* (resp. $p(\Lambda, p)$-*closed*, $\alpha(\Lambda, p)$-*closed*). The intersection of all $b(\Lambda, p)$-closed sets of X containing A is called the $b(\Lambda, p)$-*closure* of A and is denoted by $A^{b(\Lambda,p)}$.

3 Properties of weakly $b(\Lambda, p)$-open functions

We begin this section by introducing the notion of weakly $b(\Lambda, p)$-open functions.

Definition 3.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be weakly $b(\Lambda, p)$-open if $f(U) \subseteq [f(U^{(\Lambda,p)})]_{b(\Lambda,p)}$ for each (Λ, p)-open set U of X.*

Theorem 3.2. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is weakly $b(\Lambda, p)$-open;

(2) $f(A_{\theta(\Lambda,p)}) \subseteq [f(A)]_{b(\Lambda,p)}$ for every subset A of X;

(3) $[f^{-1}(B)]_{\theta(\Lambda,p)} \subseteq f^{-1}(B_{b(\Lambda,p)})$ for every subset B of Y;

(4) $f^{-1}(B^{b(\Lambda,p)}) \subseteq [f^{-1}(B)]^{\theta(\Lambda,p)}$ for every subset B of Y.

Proof. (1) $\Rightarrow$ (2): Let A be any subset of X and $x \in A_{\theta(\Lambda,p)}$. Then, there exists a (Λ, p)-open set U of X such that $x \in U \subseteq U^{(\Lambda,p)} \subseteq A$. Then, $f(x) \in f(U) \subseteq f(U^{(\Lambda,p)}) \subseteq f(A)$. Since f is weakly $b(\Lambda, p)$-open, $f(U) \subseteq [f(U^{(\Lambda,p)})]_{b(\Lambda,p)} \subseteq [f(A)]_{b(\Lambda,p)}$. This implies that $f(x) \in [f(A)]_{b(\Lambda,p)}$. Therefore, $x \in f^{-1}([f(A)]_{b(\Lambda,p)})$. Thus, $A_{\theta(\Lambda,p)} \subseteq f^{-1}([f(A)]_{b(\Lambda,p)})$ and hence $f(A_{\theta(\Lambda,p)}) \subseteq [f(A)]_{b(\Lambda,p)}$.

(2) $\Rightarrow$ (3): Let B be any subset of Y. Then by (2), $f([f^{-1}(B)]_{\theta(\Lambda,p)}) \subseteq [f(f^{-1}(B))]_{b(\Lambda,p)} \subseteq B_{b(\Lambda,p)}$. Thus, $[f^{-1}(B)]_{\theta(\Lambda,p)} \subseteq f^{-1}(B_{b(\Lambda,p)})$.

(3) $\Rightarrow$ (4): Let B be any subset of Y. Using (3), we have

$$\begin{aligned} X - [f^{-1}(B)]^{\theta(\Lambda,p)} &= [f^{-1}(Y - B)]_{\theta(\Lambda,p)} \\ &\subseteq f^{-1}([Y - B]_{b(\Lambda,p)}) = X - f^{-1}(B^{b(\Lambda,p)}) \end{aligned}$$

and hence $f^{-1}(B^{b(\Lambda,p)}) \subseteq [f^{-1}(B)]^{\theta(\Lambda,p)}$.

(4) $\Rightarrow$ (1): Let U be any (Λ, p)-open set of X. By (4),

$$f^{-1}([Y - f(U^{(\Lambda,p)})]^{b(\Lambda,p)}) \subseteq [f^{-1}(Y - f(U^{(\Lambda,p)}))]^{\theta(\Lambda,p)}.$$

Thus, $f^{-1}(Y - [f(U^{(\Lambda,p)})]_{b(\Lambda,p)}) \subseteq [X - f^{-1}(f(U^{(\Lambda,p)}))]^{\theta(\Lambda,p)} \subseteq [X - U^{(\Lambda,p)}]^{\theta(\Lambda,p)}$ and hence $U \subseteq [U^{(\Lambda,p)}]_{\theta(\Lambda,p)} \subseteq f^{-1}([f(U^{(\Lambda,p)})]_{b(\Lambda,p)})$. Therefore, $f(U) \subseteq [f(U^{(\Lambda,p)})]_{b(\Lambda,p)}$. This shows that f is weakly $b(\Lambda, p)$-open. $\square$

Theorem 3.3. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is weakly $b(\Lambda, p)$-open;

(2) for each $x \in X$ and each (Λ, p)-open set U of X containing x, there exists a $b(\Lambda, p)$-open set V of Y containing $f(x)$ such that $V \subseteq f(U^{(\Lambda,p)})$.

Proof. (1) $\Rightarrow$ (2): Let U be any (Λ,p)-open set of X with $x \in U$. Since f is weakly $b(\Lambda,p)$-open, $f(x) \in f(U) \subseteq [f(U^{(\Lambda,p)})]_{b(\Lambda,p)}$. Let $V = [f(U^{(\Lambda,p)})]_{b(\Lambda,p)}$. Then, V is $b(\Lambda,p)$-open and $f(x) \in V \subseteq f(U^{(\Lambda,p)})$.

(2) $\Rightarrow$ (1): Let U be any (Λ,p)-open set of X and $y \in f(U)$. It follows from (2) that $V \subseteq f(U^{(\Lambda,p)})$ for some $b(\Lambda,p)$-open set V of Y containing y. Thus, $y \in V \subseteq [f(U^{(\Lambda,p)})]_{b(\Lambda,p)}$ and hence $f(U) \subseteq [f(U^{(\Lambda,p)})]_{b(\Lambda,p)}$. This shows that f is weakly $b(\Lambda,p)$-open. □

Theorem 3.4. *For a function $f : (X,\tau) \to (Y,\sigma)$, the following properties are equivalent:*

(1) f is weakly $b(\Lambda,p)$-open;

(2) $[f(K_{(\Lambda,p)})]^{b(\Lambda,p)} \subseteq f(K)$ for each (Λ,p)-closed set K of X;

(3) $[f(U)]^{b(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ for each (Λ,p)-open set U of X.

Proof. (1) $\Rightarrow$ (2): Let K be any (Λ,p)-closed set of X. Then, we have $f(X-K) = Y - f(K) \subseteq [f([X-K]^{(\Lambda,p)})]_{b(\Lambda,p)}$ and so

$$Y - f(K) \subseteq Y - [f(K_{(\Lambda,p)})]^{b(\Lambda,p)}.$$

Thus, $[f(K_{(\Lambda,p)})]^{b(\Lambda,p)} \subseteq f(K)$.

(2) $\Rightarrow$ (3): Let U be any (Λ,p)-open set of X. Since $U^{(\Lambda,p)}$ is a (Λ,p)-closed set and $U \subseteq [U^{(\Lambda,p)}]_{(\Lambda,p)}$, by (2) we have

$$[f(U)]^{b(\Lambda,p)} \subseteq [f([U^{(\Lambda,p)}]_{(\Lambda,p)})]^{b(\Lambda,p)} \subseteq f(U^{(\Lambda,p)}).$$

(3) $\Rightarrow$ (1): Let U be any (Λ,p)-open set of X. Using (3), we have

$$\begin{aligned} Y - [f(U^{(\Lambda,p)})]_{b(\Lambda,p)} &= [Y - f(U^{(\Lambda,p)})]^{b(\Lambda,p)} \\ &= [f(X - U^{(\Lambda,p)})]^{b(\Lambda,p)} \\ &\subseteq f([X - U^{(\Lambda,p)}]^{(\Lambda,p)}) \\ &= f(X - [U^{(\Lambda,p)}]_{(\Lambda,p)}) \subseteq f(X-U) = Y - f(U) \end{aligned}$$

and hence $f(U) \subseteq [f(U^{(\Lambda,p)})]_{b(\Lambda,p)}$. This shows that f is weakly $b(\Lambda,p)$-open. □

The proof of the following theorem is straightforward and thus is omitted.

Theorem 3.5. *For a function $f : (X,\tau) \to (Y,\sigma)$, the following properties are equivalent:*

(1) f *is weakly* $b(\Lambda, p)$*-open;*

(2) $f(U) \subseteq [f(U^{(\Lambda,p)})]_{b(\Lambda,p)}$ *for each* $p(\Lambda, p)$*-open set* U *of* X*;*

(3) $f(U) \subseteq [f(U^{(\Lambda,p)})]_{b(\Lambda,p)}$ *for each* $\alpha(\Lambda, p)$*-open set* U *of* X*;*

(4) $f([U^{(\Lambda,p)}]_{(\Lambda,p)}) \subseteq [f(U^{(\Lambda,p)})]_{b(\Lambda,p)}$ *for each* (Λ, p)*-open set* U *of* X*;*

(5) $f(K_{(\Lambda,p)}) \subseteq [f(K)]_{b(\Lambda,p)}$ *for each* (Λ, p)*-closed set* K *of* X*.*

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