Characterizations of weakly $\delta(\Lambda, p)$-closed functions

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(Received July 11, 2023, Accepted November 6, 2023,
Published November 10, 2023)

Abstract
This paper is concerned with the concept of weakly $\delta(\Lambda, p)$-closed functions. Moreover, several characterizations of weakly $\delta(\Lambda, p)$-closed functions are established.

1 Introduction


Key words and phrases: $\delta(\Lambda, p)$-open set, $\delta(\Lambda, p)$-closed function.
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AMS (MOS) Subject Classifications: 54A05, 54C10
ISSN 1814-0432, 2024, http://ijmcs.future-in-tech.net
of weakly $\delta$-closedness as a new generalization of $\delta$-closedness and obtained several characterizations of weakly $\delta$-closed functions. In [3], the present authors investigated some properties of $(\Lambda, sp)$-closed sets and $(\Lambda, sp)$-open sets. Boonpok and Viriyapong [4] introduced and investigated the concepts of $(\Lambda, p)$-open sets and $(\Lambda, p)$-closed sets. Quite recently, Boonpok and Thongmoon [2] introduced the notions of $\delta(\Lambda, p)$-closed sets and $\delta(\Lambda, p)$-open sets in topological spaces. In this paper, we introduce the concept of weakly $\delta(\Lambda, p)$-closed functions. Moreover, we investigate some characterizations of weakly $\delta(\Lambda, p)$-closed functions.

2 Preliminaries

Throughout the present paper, unless explicitly stated, spaces $(X, \tau)$ and $(Y, \sigma)$ (or simply $X$ and $Y$) always mean topological spaces on which no separation axioms are assumed. For a subset $A$ of a topological space $(X, \tau)$, $\text{Cl}(A)$ and $\text{Int}(A)$, represent the closure and the interior of $A$, respectively. A subset $A$ of a topological space $(X, \tau)$ is said to be preopen [9] if $A \subseteq \text{Int}(\text{Cl}(A))$. The complement of a preopen set is called preclosed. The family of all preopen sets of a topological space $(X, \tau)$ is denoted by $PO(X, \tau)$. A subset $\Lambda_p(A)$ [7] is defined as follows:

$$\Lambda_p(A) = \cap\{U \mid A \subseteq U, U \in PO(X, \tau)\}.$$ 

A subset $A$ of a topological space $(X, \tau)$ is called a $\Lambda_p$-set [4] (pre-$\Lambda$-set [7]) if $A = \Lambda_p(A)$. A subset $A$ of a topological space $(X, \tau)$ is called $(\Lambda, p)$-closed [4] if $A = T \cap C$, where $T$ is a $\Lambda_p$-set and $C$ is a preclosed set. The complement of a $(\Lambda, p)$-closed set is called $(\Lambda, p)$-open. A point $x \in X$ is called a $(\Lambda, p)$-cluster point [4] of $A$ if $A \cap U \neq \emptyset$ for every $(\Lambda, p)$-open set $U$ of $X$ containing $x$. The set of all $(\Lambda, p)$-cluster points of $A$ is called the $(\Lambda, p)$-closure [4] of $A$ and is denoted by $A^{(\Lambda, p)}$. The union of all $(\Lambda, p)$-open sets of $X$ contained in $A$ is called the $(\Lambda, p)$-interior [4] of $A$ and is denoted by $A_{(\Lambda, p)}$. A subset $A$ of a topological space $(X, \tau)$ is called $p(\Lambda, p)$-open [4] (resp. $\alpha(\Lambda, p)$-open [14]) if $A \subseteq [A^{(\Lambda, p)}]_{(\Lambda, p)}$ (resp. $A \subseteq [[A_{(\Lambda, p)}]^{(\Lambda, p)}]_{(\Lambda, p)}$). The complement of a $p(\Lambda, p)$-open (resp. $\alpha(\Lambda, p)$-open) set is called $p(\Lambda, p)$-closed (resp. $\alpha(\Lambda, p)$-closed). A subset $A$ of a topological space $(X, \tau)$ is called $r(\Lambda, p)$-open [4] if $A = [A^{(\Lambda, p)}]_{(\Lambda, p)}$. Let $A$ be a subset of a topological space $(X, \tau)$. A point $x$ of $X$ is called a $\delta(\Lambda, p)$-cluster point [2] of $A$ if $A \cap [V^{(\Lambda, p)}]_{(\Lambda, p)} \neq \emptyset$ for every $(\Lambda, p)$-open set $V$ of $X$ containing $x$. The set of all $\delta(\Lambda, p)$-cluster points of $A$ is called the $\delta(\Lambda, p)$-closure [2] of $A$ and is denoted by $A^{\delta(\Lambda, p)}$. If $A = A^{\delta(\Lambda, p)}$, then $A$ is said to be $\delta(\Lambda, p)$-closed [2]. The complement of a $\delta(\Lambda, p)$-closed
set is said to be $\delta(\Lambda, p)$-open. The union of all $\delta(\Lambda, p)$-open sets contained in $A$ is called the $\delta(\Lambda, p)$-interior [2] of $A$ and is denoted by $A_{\delta(\Lambda, p)}$.

3 Some characterizations of weakly $\delta(\Lambda, p)$-closed functions

We begin this section by introducing the concept of weakly $\delta(\Lambda, p)$-open functions.

**Definition 3.1.** A function $f : (X, \tau) \to (Y, \sigma)$ is called weakly $\delta(\Lambda, p)$-closed if $[f(K_{(\Lambda, p)})]^{\delta(\Lambda, p)} \subseteq f(K)$ for every $(\Lambda, p)$-closed set $K$ of $X$.

**Theorem 3.2.** For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:

1. $f$ is weakly $\delta(\Lambda, p)$-closed;
2. $[f(U)]^{\delta(\Lambda, p)} \subseteq f(U^{(\Lambda, p)})$ for every $(\Lambda, p)$-open set $U$ of $X$;
3. $[f(U)]^{\delta(\Lambda, p)} \subseteq f(U^{(\Lambda, p)})$ for every $r(\Lambda, p)$-open set $U$ of $X$;
4. $[f(K_{(\Lambda, p)})]^{\delta(\Lambda, p)} \subseteq f(K)$ for every $p(\Lambda, p)$-closed set $K$ of $X$;
5. $[f(K_{(\Lambda, p)})]^{\delta(\Lambda, p)} \subseteq f(K)$ for every $a(\Lambda, p)$-closed set $K$ of $X$;
6. $[f([A^{(\Lambda, p)}]_{(\Lambda, p)})]^{\delta(\Lambda, p)} \subseteq f(A^{(\Lambda, p)})$ for every subset $A$ of $X$;
7. $[f([A^{\delta(\Lambda, p)}]_{(\Lambda, p)})]^{\delta(\Lambda, p)} \subseteq f(A^{\delta(\Lambda, p)})$ for every subset $A$ of $X$;
8. $[f(U)]^{\delta(\Lambda, p)} \subseteq f(U^{(\Lambda, p)})$ for every $p(\Lambda, p)$-open set $U$ of $X$.

**Proof.** (1) $\Rightarrow$ (2): Let $U$ be any $(\Lambda, p)$-open set of $X$. Then, $[f(U)]^{\delta(\Lambda, p)} = [f(U^{(\Lambda, p)})]^{\delta(\Lambda, p)} \subseteq f([U^{(\Lambda, p)}]_{(\Lambda, p)})^{\delta(\Lambda, p)} \subseteq f(U^{(\Lambda, p)})$.

(2) $\Rightarrow$ (1): Let $K$ be any $(\Lambda, p)$-closed set of $X$. Then, $[f(K_{(\Lambda, p)})]^{\delta(\Lambda, p)} \subseteq f([K_{(\Lambda, p)}]^{\delta(\Lambda, p)})^{\delta(\Lambda, p)} = F(K)$.

It is clear that: (1) $\Rightarrow$ (3) $\Rightarrow$ (4) $\Rightarrow$ (5) $\Rightarrow$ (1), (1) $\Rightarrow$ (6) $\Rightarrow$ (8) $\Rightarrow$ (1) and (1) $\Rightarrow$ (7).

(7) $\Rightarrow$ (8): This is obvious since $U^{\delta(\Lambda, p)} = U^{(\Lambda, p)}$ for every $p(\Lambda, p)$-open set $U$ of $X$.

**Definition 3.3.** [11] A topological space $(X, \tau)$ is said to be $(\Lambda, p)$-regular if for each $(\Lambda, p)$-closed set $F$ and each point $x \in X - F$, there exist disjoint $(\Lambda, p)$-open sets $U$ and $V$ such that $x \in U$ and $F \subseteq V$. 

\[ \]
Theorem 3.4. Let \((Y, \sigma)\) be a \((\Lambda, p)\)-regular space. For a function \(f: (X, \tau) \to (Y, \sigma)\), the following properties are equivalent:

1. \(f\) is weakly \(\delta(\Lambda, p)\)-closed;
2. \([f(U)]^{\delta(\Lambda, p)} \subseteq f(U^{(\Lambda, p)})\) for every \(r(\Lambda, p)\)-open set \(U\) of \(X\);
3. for each subset \(B\) of \(Y\) and each \((\Lambda, p)\)-open set \(U\) of \(X\) with \(f^{-1}(B) \subseteq U\), there exists a \(\delta(\Lambda, p)\)-open set \(V\) of \(Y\) such that \(B \subseteq V\) and \(f^{-1}(V) \subseteq U^{(\Lambda, p)}\);
4. for each point \(y \in Y\) and each \((\Lambda, p)\)-open set \(U\) of \(X\) with \(f^{-1}(y) \subseteq U\), there exists a \(\delta(\Lambda, p)\)-open set \(V\) of \(Y\) containing \(y\) such that \(f^{-1}(V) \subseteq U^{(\Lambda, p)}\).

Proof. \((1) \Rightarrow (2)\): This is obvious.

\((2) \Rightarrow (3)\): Let \(B\) be any subset of \(Y\) and \(U\) be \((\Lambda, p)\)-open in \(X\) with \(f^{-1}(B) \subseteq U\). Then, \(f^{-1}(B) \cap [X - U^{(\Lambda, p)}]^{(\Lambda, p)} = \emptyset\) and hence

\[B \cap f([X - U^{(\Lambda, p)}]^{(\Lambda, p)}) = \emptyset.\]

Since \(X - U^{(\Lambda, p)}\) is \(r(\Lambda, p)\)-open, \(B\) \(\cap [f(X - U^{(\Lambda, p)})]^{\delta(\Lambda, p)} = \emptyset\) by \((2)\). Let \(V = Y - [f(X - U^{(\Lambda, p)})]^{\delta(\Lambda, p)}\). Then, \(V\) is \(\delta(\Lambda, p)\)-open such that \(B \subseteq V\) and \(f^{-1}(V) \subseteq X - f^{-1}([f(X - U^{(\Lambda, p)})]^{\delta(\Lambda, p)}) \subseteq X - f^{-1}(f(X - U^{(\Lambda, p)})) \subseteq U^{(\Lambda, p)}\).

\((3) \Rightarrow (4)\): This is obvious.

\((4) \Rightarrow (1)\): Let \(K\) be any \((\Lambda, p)\)-closed set of \(X\) and \(y \in Y - f(K)\). Since \(f^{-1}(y) \subseteq X - K\), there exists a \(\delta(\Lambda, p)\)-open set \(V\) of \(Y\) such that \(y \in V\) and \(f^{-1}(V) \subseteq [Y - K]^{(\Lambda, p)} = X - K^{(\Lambda, p)}\) by \((4)\). Then, we have \(V \cap f(K^{(\Lambda, p)}) = \emptyset\) and hence \(y \in Y - [f(K^{(\Lambda, p)})]^{\delta(\Lambda, p)}\). Thus, \([f(K^{(\Lambda, p)})]^{\delta(\Lambda, p)} \subseteq f(K)\). This shows that \(f\) is weakly \(\delta(\Lambda, p)\)-closed.

Acknowledgment. This research project was financially supported by Mahasarakham University.

References


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