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Characterizations of weakly $\delta(\Lambda, p)$-closed functions

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Abstract

This paper is concerned with the concept of weakly $\delta(\Lambda, p)$-closed functions. Moreover, several characterizations of weakly $\delta(\Lambda, p)$-closed functions are established.

1 Introduction

In 1982, Mashhour et al. [9] introduced and studied the notion of preclosed functions. Noiri [10] introduced and investigated the concept of semi-closed functions. In 1983, Mashhour et al. [8] studied some characterizations of α-closed functions. El-Monsef et al. [1] introduced and investigated the notions of β-closed functions. In 1984, Rose [13] introduced and studied the notions of weakly open functions and almost open functions. Rose and Janković [12] investigated some of the fundamental properties of weakly closed functions. In 2004, Caldas and Navalagi [6] introduced two new classes of functions called weakly preopen functions and weakly preclosed functions as generalization of weak openness and weak closedness due to [13] and [12], respectively. In 2008, Caldas and Navalagi [5] introduced and discussed the notion

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of weakly δ-closedness as a new generalization of δ-closedness and obtained several characterizations of weakly δ-closed functions. In [3], the present authors investigated some properties of (Λ, sp)-closed sets and (Λ, sp)-open sets. Boonpok and Viriyapong [4] introduced and investigated the concepts of (Λ, p)-open sets and (Λ, p)-closed sets. Quite recently, Boonpok and Thongmoon [2] introduced the notions of $\delta(\Lambda, p)$-closed sets and $\delta(\Lambda, p)$-open sets in topological spaces. In this paper, we introduce the concept of weakly $\delta(\Lambda, p)$-closed functions. Moreover, we investigate some characterizations of weakly $\delta(\Lambda, p)$-closed functions.

2 Preliminaries

Throughout the present paper, unless explicitly stated, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed. For a subset A of a topological space (X, τ), $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, represent the closure and the interior of A, respectively. A subset A of a topological space (X, τ) is said to be *preopen* [9] if $A \subseteq \mathrm{Int}(\mathrm{Cl}(A))$. The complement of a preopen set is called *preclosed*. The family of all preopen sets of a topological space (X, τ) is denoted by $PO(X, \tau)$. A subset $\Lambda_p(A)$ [7] is defined as follows:
$\Lambda_p(A) = \cap\{U \mid A \subseteq U, U \in PO(X, \tau)\}$. A subset A of a topological space (X, τ) is called a Λ_p-*set* [4] (*pre*-Λ-*set* [7]) if $A = \Lambda_p(A)$. A subset A of a topological space (X, τ) is called (Λ, p)-*closed* [4] if $A = T \cap C$, where T is a Λ_p-set and C is a preclosed set. The complement of a (Λ, p)-closed set is called (Λ, p)-*open*. A point $x \in X$ is called a (Λ, p)-*cluster point* [4] of A if $A \cap U \neq \emptyset$ for every (Λ, p)-open set U of X containing x. The set of all (Λ, p)-cluster points of A is called the (Λ, p)-*closure* [4] of A and is denoted by $A^{(\Lambda,p)}$. The union of all (Λ, p)-open sets of X contained in A is called the (Λ, p)-*interior* [4] of A and is denoted by $A_{(\Lambda,p)}$. A subset A of a topological space (X, τ) is called $p(\Lambda, p)$-*open* [4] (resp. $\alpha(\Lambda, p)$-*open* [14]) if $A \subseteq [A^{(\Lambda,p)}]_{(\Lambda,p)}$ (resp. $A \subseteq [[A_{(\Lambda,p)}]^{(\Lambda,p)}]_{(\Lambda,p)}$). The complement of a $p(\Lambda, p)$-open (resp. $\alpha(\Lambda, p)$-open) set is called $p(\Lambda, p)$-*closed* (resp. $\alpha(\Lambda, p)$-*closed*). A subset A of a topological space (X, τ) is called $r(\Lambda, p)$-*open* [4] if $A = [A^{(\Lambda,p)}]_{(\Lambda,p)}$. Let A be a subset of a topological space (X, τ). A point x of X is called a $\delta(\Lambda, p)$-*cluster point* [2] of A if $A \cap [V^{(\Lambda,p)}]_{(\Lambda,p)} \neq \emptyset$ for every (Λ, p)-open set V of X containing x. The set of all $\delta(\Lambda, p)$-cluster points of A is called the $\delta(\Lambda, p)$-*closure* [2] of A and is denoted by $A^{\delta(\Lambda,p)}$. If $A = A^{\delta(\Lambda,p)}$, then A is said to be $\delta(\Lambda, p)$-*closed* [2]. The complement of a $\delta(\Lambda, p)$-closed

set is said to be $\delta(\Lambda, p)$-*open*. The union of all $\delta(\Lambda, p)$-open sets contained in A is called the $\delta(\Lambda, p)$-*interior* [2] of A and is denoted by $A_{\delta(\Lambda,p)}$.

3 Some characterizations of weakly $\delta(\Lambda, p)$-closed functions

We begin this section by introducing the concept of weakly $\delta(\Lambda, p)$-open functions.

Definition 3.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is called weakly $\delta(\Lambda, p)$-closed if $[f(K_{(\Lambda,p)})]^{\delta(\Lambda,p)} \subseteq f(K)$ for every (Λ, p)-closed set K of X.*

Theorem 3.2. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is weakly $\delta(\Lambda, p)$-closed;

(2) $[f(U)]^{\delta(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ for every (Λ, p)-open set U of X;

(3) $[f(U)]^{\delta(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ for every $r(\Lambda, p)$-open set U of X;

(4) $[f(K_{(\Lambda,p)})]^{\delta(\Lambda,p)} \subseteq f(K)$ for every $p(\Lambda, p)$-closed set K of X;

(5) $[f(K_{(\Lambda,p)})]^{\delta(\Lambda,p)} \subseteq f(K)$ for every $\alpha(\Lambda, p)$-closed set K of X;

(6) $[f([A^{(\Lambda,p)}]_{(\Lambda,p)})]^{\delta(\Lambda,p)} \subseteq f(A^{(\Lambda,p)})$ for every subset A of X;

(7) $[f([A^{\delta(\Lambda,p)}]_{(\Lambda,p)})]^{\delta(\Lambda,p)} \subseteq f(A^{\delta(\Lambda,p)})$ for every subset A of X;

(8) $[f(U)]^{\delta(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ for every $p(\Lambda, p)$-open set U of X.

Proof. (1) $\Rightarrow$ (2): Let U be any (Λ, p)-open set of X. Then, $[f(U)]^{\delta(\Lambda,p)} = [f(U_{(\Lambda,p)})]^{\delta(\Lambda,p)} \subseteq [f([U^{(\Lambda,p)}]_{(\Lambda,p)})]^{\delta(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$.

(2) $\Rightarrow$ (1): Let K be any (Λ, p)-closed set of X. Then, $[f(K_{(\Lambda,p)})]^{\delta(\Lambda,p)} \subseteq f([K_{(\Lambda,p)}]^{(\Lambda,p)}) \subseteq f(K^{(\Lambda,p)}) = F(K)$.

It is clear that: (1) $\Rightarrow$ (3) $\Rightarrow$ (4) $\Rightarrow$ (5) $\Rightarrow$ (1), (1) $\Rightarrow$ (6) $\Rightarrow$ (8) $\Rightarrow$ (1) and (1) $\Rightarrow$ (7).

(7) $\Rightarrow$ (8): This is obvious since $U^{\delta(\Lambda,p)} = U^{(\Lambda,p)}$ for every $p(\Lambda, p)$-open set U of X. $\square$

Definition 3.3. [11] *A topological space (X, τ) is said to be (Λ, p)-regular if for each (Λ, p)-closed set F and each point $x \in X - F$, there exist disjoint (Λ, p)-open sets U and V such that $x \in U$ and $F \subseteq V$.*

Theorem 3.4. *Let (Y, σ) be a (Λ, p)-regular space. For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is weakly $\delta(\Lambda, p)$-closed;

(2) $[f(U)]^{\delta(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ for every $r(\Lambda, p)$-open set U of X;

(3) for each subset B of Y and each (Λ, p)-open set U of X with $f^{-1}(B) \subseteq U$, there exists a $\delta(\Lambda, p)$-open set V of Y such that $B \subseteq V$ and $f^{-1}(V) \subseteq U^{(\Lambda,p)}$;

(4) for each point $y \in Y$ and each (Λ, p)-open set U of X with $f^{-1}(y) \subseteq U$, there exists a $\delta(\Lambda, p)$-open set V of Y containing y such that $f^{-1}(V) \subseteq U^{(\Lambda,p)}$.

Proof. (1) $\Rightarrow$ (2): This is obvious.

(2) $\Rightarrow$ (3): Let B be any subset of Y and U be (Λ, p)-open in X with $f^{-1}(B) \subseteq U$. Then, $f^{-1}(B) \cap [X - U^{(\Lambda,p)}]^{(\Lambda,p)} = \emptyset$ and hence

$$B \cap f([X - U^{(\Lambda,p)}]^{(\Lambda,p)}) = \emptyset.$$

Since $X - U^{(\Lambda,p)}$ is $r(\Lambda, p)$-open, $B \cap [f(X - U^{(\Lambda,p)})]^{\delta(\Lambda,p)} = \emptyset$ by (2). Let $V = Y - [f(X - U^{(\Lambda,p)})]^{\delta(\Lambda,p)}$. Then, V is $\delta(\Lambda, p)$-open such that $B \subseteq V$ and $f^{-1}(V) \subseteq X - f^{-1}([f(X - U^{(\Lambda,p)})]^{\delta(\Lambda,p)}) \subseteq X - f^{-1}(f(X - U^{(\Lambda,p)})) \subseteq U^{(\Lambda,p)}$.

(3) $\Rightarrow$ (4): This is obvious.

(4) $\Rightarrow$ (1): Let K be any (Λ, p)-closed set of X and $y \in Y - f(K)$. Since $f^{-1}(y) \subseteq X - K$, there exists a $\delta(\Lambda, p)$-open set V of Y such that $y \in V$ and $f^{-1}(V) \subseteq [Y - K]^{(\Lambda,p)} = X - K_{(\Lambda,p)}$ by (4). Then, we have $V \cap f(K_{(\Lambda,p)}) = \emptyset$ and hence $y \in Y - [f(K_{(\Lambda,p)})]^{\delta(\Lambda,p)}$. Thus, $[f(K_{(\Lambda,p)})]^{\delta(\Lambda,p)} \subseteq f(K)$. This shows that f is weakly $\delta(\Lambda, p)$-closed. □

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