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β-multiplication Fuzzy q-ideals of RG-algebra

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Abstract

In this paper, we introduce the concepts of β-multiplication fuzzy q-ideals of RG-algebras and we study several basic properties of it.

1 Introduction

We present the concept of β-multiplication fuzzy q-ideals of RG-algebras along with related properties. Mahadi, Hameed, and Malik [1, 2] introduced the concept of magnified translation of intuitionistic fuzzy AT-ideals on AT-algebras. Kareem et al. [3, 4] introduced multipliers of an AT-algebra. Patthanangkoor [5] introduced the concept of homomorphism of

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RG-algebras and investigated related properties. R.A.K. Omar [6] introduced RG-algebras, RG-ideals, and RG-subalgebras and studyied their relationships. Jasim and Hameed [7] introduced fuzzy RG-subalgebras and fuzzy RG-ideals of RG-algebras which were further further explored by Jasim. Abrahem and Abed [8, 9] studied fuzzy RG-ideals of RG-algebras, focusing on properties like homomorphism image and inverse image. Al-Talal and Hameed [10] examined new concepts of ideals and q-ideals of RG-algebras with homomorphism RG-algebras.

2 Preliminaries

We now give some definitions and preliminary results that bare needed in the later sections.

Definition 2.1. *[6] An algebra $(\partial; *, e)$ is called an RG-algebra if $\forall\ \rho, \sigma,\ \tau \in \partial$, the following axioms are satisfied:*

i. $\rho * e = \rho$,

ii. $\rho * \sigma = (\rho * \tau) * (\sigma * \tau)$,

iii. $\rho * \sigma = \sigma * \rho = e$ *imply* $\rho = \sigma$.

Definition 2.2. *[6] Let $(\partial; *, e)$ be an RG-algebra. A nonempty subset $\mathtt{C}$ of ∂ is called an q-ideal of ∂ if $\forall \rho, \sigma \in \partial$, we have*

i. $e \in \mathtt{C}$,

ii. $(\rho * \sigma) * \tau \in \mathtt{C}$ *and* $\sigma \in \mathtt{C}$ *imply* $\rho * \tau \in \mathtt{C}$.

3 β-multiplication of Fuzzy q-ideals of RG-algebra

We define the notion of β-multiplication of fuzzy q-ideals and we study some of the relations and results of β-multiplication of fuzzy q-ideals of RG-algebra.

Definition 3.1. *For a fuzzy subset π of a set $(\partial; *, e)$, β ((0,1], and $t \in Im(\mu)$ with $t \leq \beta$, we have $\mathbf{U}_\beta(\pi; t) = \{\sigma \in \partial | \pi(\sigma) \geq t/\beta\}$.*

Definition 3.2. *Let π be a fuzzy subset of a set $(\partial; *, e)$ and let $\beta \in (0, 1]$. A mapping $\pi^M_\beta : \partial \in [0, 1]$ is called a β-multiplication of π if $\pi^M_\beta(\sigma) = \beta.\ \pi(\sigma)$, for all $\sigma \in \partial$.*

Definition 3.3. *Let $(\partial; *, e)$ be a nonempty set. A β-magnified fuzzy subset π of ∂ is called a β-magnified FI of ∂ (MFq-I) if $\forall\sigma,\ \rho \in \partial$, it satisfies the following conditions:*

i. $\pi_\beta^M(e) \geq \pi_\beta^M\ (\sigma)$,

ii. $\pi_\beta^M(\rho)\ \geq\ min\ \ \pi_\beta^M(\sigma * \rho), \pi_\beta^M(\sigma)$.

Definition 3.4. *Let $(\partial; *, e)$ be nonempty set. A β-multiplication fuzzy subset π of ∂ is called a β-multiplication Fq-I of ∂ (MFq-I) if $\forall\sigma,\ \rho,\ \varepsilon \in \partial$, it satisfies the following conditions:*

i. $\pi_\beta^M(e)\ \geq\ \pi_\beta^M\ (\sigma)$,

ii. $\pi_\beta^M(\sigma * \varepsilon) \geq \min\{\pi_\beta^M((\sigma * \rho) * \varepsilon), \pi_\beta^M(\rho)\}$.

Theorem 3.5. *Let π be a fuzzy subset of AN RG-algebra $(\partial; *, e)$ and let$\beta \in$ $)0, \frac{1}{\pi(e)}] \ni \pi(e) \neq 0$. Then π is a Fq-I of $\partial \Longleftrightarrow \pi_\beta^M$ is a Fq-I of ∂*

Proof.
($\Rightarrow$) Assume that π is a fuzzy q-ideal of ∂, and $\beta \in)0, \frac{1}{\pi(e)}] \ni \pi(e) \neq 0$.

Since $\pi(e)\ \geq\ \pi\ (\sigma), \pi_\beta^M\ (e) = \beta$. Also, $\pi\ (e) \geq\ \beta.\ \pi\ (\sigma) =\ \pi_\beta^M\ \ (\sigma)$, for all $\sigma \in \partial$. $\forall\sigma,\ \rho,\ \varepsilon \in \partial$ and so $\pi\ (\sigma * \varepsilon)\ \geq \min \pi((\sigma * \rho) * \varepsilon),\ \pi(\rho)$.

Thus

$$\pi_\beta^M\ (\sigma * \varepsilon) = \beta\ .\pi\ (\sigma * \varepsilon) \geq\ \ \beta\ .\min\left\{\pi((\sigma * \rho) * \varepsilon), \pi\ (\rho)\right\}$$

$$= \min\{\beta.\pi((\sigma * \rho) * \varepsilon), \beta.\pi(\rho)\} = \min\left\{\pi_\beta^M\ \ ((\sigma * \rho) * \varepsilon), \pi_\beta^M\ (\rho)\right\}$$

and so

$$\pi_\beta^M\ \ (\sigma * \varepsilon)\ \ \geq \min \pi_\beta^M\ \ ((\sigma * \rho) * \varepsilon), \pi_\beta^M(\rho).$$

As a result, π_β^M is a Fq-I of ∂.

($\Leftarrow$) π_β^M is a Fq-I of $\partial\beta$, where $\beta \in\ )0, \frac{1}{\pi(e)}] \ni \pi(e) \neq 0$.

Since $\pi_\beta^M\ (e)\ \ \geq \pi_\beta^M\ \ (\sigma)\ , \pi(e) \geq\ \pi\ (\sigma)$, for all $\sigma \in \partial.\forall\sigma,\ \rho,\ \varepsilon \in \partial$ and so

$$\beta.\ \pi\ (\sigma * \varepsilon) =\ \pi_\beta^M\ (\sigma * \varepsilon) \geq \min\left\{\pi_\beta^M((\sigma * \rho) * \varepsilon), \pi_\beta^M\ (\rho)\right\}$$

$$= \min \beta.\pi((\sigma * \rho) * \varepsilon), \beta.\pi(y) = \beta \min\left\{\pi((\sigma * \rho) * \varepsilon), \pi\ (\rho)\right\}.$$

Therefore,

$$\pi(\sigma * \varepsilon)\ \ \geq \min \pi((\sigma * \rho) * \varepsilon), \pi(\rho)$$

. Consequently, is a Fq-I of ∂. ■

Proposition 3.6. *Let π be a fuzzy subset of an RG-algebra $(\partial;\ *, e)$ and let $\beta \in)0, \frac{1}{\pi(e)}] \ni \pi(e) \neq 0$. If π_β^M is a Fq-I of ∂, then $U_\beta(\pi;t)$ is a q-I of ∂, for any $t \in [0,1]$ with $t \leq \beta$.*

Proof.
π_β^M is a Fq-I by Theroem (3.5) $\Rightarrow \pi$ is a Fq-I $\Rightarrow$for all $t \in [0,1]$ with $t \leq \beta$. $\forall \sigma, \rho, \varepsilon \in\ U_\beta(\pi;t) \Rightarrow \pi((\sigma * \rho) * \varepsilon) \geq t/\beta$ and $\pi(\rho) \geq t/\beta \Rightarrow \min\{\pi)((\sigma * \rho) * \varepsilon), \pi(\rho)\} \geq t/\beta$, since π is Fq-I $\Rightarrow \pi(\sigma * \varepsilon) \geq \min\{\pi((\sigma * \rho) * \varepsilon), \pi(\rho)\} \geq t/\beta \Rightarrow \sigma * \varepsilon \in\ U_\beta(\pi;t)$. Hence $U_\beta(\pi;t)$ is a q-I of ∂. ■

Proposition 3.7. *Let π be a fuzzy subset of an RG-algebra $(\partial;\ *, e)$ and let $\beta \in)0, \frac{1}{\pi(e)}] \ni \pi(e) \neq 0$. If $U_\beta(\pi;t)$ is a q-I of ∂, for all $t \in [0,1]$ with $t \leq \beta$, then π_β^M is a Fq-I of ∂.*

Proof.
Suppose, to get a contradiction, that π_β^M is not Fq-I of ∂. Since $U_\beta(\pi;t)$ is a q-I of ∂, let $\sigma, \rho, \varepsilon \in U_\beta(\mu;t)$. That means $\pi((\sigma * \rho) * \varepsilon) \in t/\beta$ and $\pi(\rho) \geq t/\beta$ but $\pi(\sigma*\varepsilon) < t/\beta$ and so $\sigma*\varepsilon\ (U_\beta(\pi;t).C!$. Therefore, $\pi_\beta^M(\sigma*\varepsilon) \geq \min \pi_\beta^M((\sigma * \rho) * \varepsilon), \pi_\beta^M(\rho)$, which is a contradiction. Consequently, π_β^M is a Fq-I of ∂. ■

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