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Reliability Bayesian estimation for generalized pareto two stress-strength models

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Abstract

Assuming that the two variables X and Y are independent generalized Pareto distributed with a known common scale parameter λ, we address a Bayesian analysis for two stress-strength model reliabilities R_1 and R_2. Three various loss functions (Squared, De-Groot, and Precautionary) are used under doubly type II censored data in order to examine the impact of the two unknown shape parameters, α and θ, respectively, on the reliability functions. We compare the three estimators and apply the Mean squared error (MSE) criteria taking different sample sizes using Monte Carlo simulations to compare the different proposed methods. Finally, using the final results from the simulation, we show that the best achievement is the Precautionary Loss Function.

1 Introduction

The generalized Pareto distribution (GPD) is an important distribution in statistics, which is widely used in the fields of finance and engineering. GPD

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was introduced by Pickands [1] and has since been further studied by Davison [2], Castillo [3] and Rezaei et al. [4] who derived the maximum likelihood estimators, Bayes estimators and some confidence intervals for stress-strength reliability based on progressive Type II censoring schemes. El-Sagheer [5] dealt with the Bayesian point prediction for the GPD based on general progressive Type II censored sample. Karam and Jbur [6] considered the Bayesian analysis of the unknown parameters under different priors and loss functions and doubly Type II censored samples.

From the properties of the Generalized Pareto distribution, if the random variable Xhas a generalized Pareto distribution, then the conditional distribution of $X \ - t$ given by $X \ \geq \ t$ is also generalized Pareto, with the same value of k. In addition to the failure rate $r\,(x) = f(x)/\,(1 - F(x))$ is given by $r(x) = 1/(\alpha - kx)$ and is monotonic in x, decreasing if $k < 0$, constant if $k \ = \ 0$, and increasing if $k > 0$. The cumulative distribution function cdf and the probability density function pdf of this distribution (GP) with twoparameters is defined as [7]:

$$F\,(x) = 1 - e^{-\alpha ln_(1+\lambda x)} = 1 - (1 + \lambda x)^{-\alpha} \tag{1.1}$$

And,

$$\begin{aligned} f\,(x) &= \alpha\lambda\,(1 + \lambda x)^{-(\alpha+1)} \\ &= \alpha\lambda e^{-(\alpha+1)ln_{(1+\lambda x)}} \end{aligned} \tag{1.2}$$

where $x \geq 0$, the shape and scale parameter $\alpha, \lambda > 0$. The system will be described for reliability estimation $R = P[Y < X]$, where the random variables X and Y are the independent Pareto distributions with different parameters. The mathematical expression has derived for the Multicomponent model reliabilities R_1 and R_2, when X is a strength for the $k$components that are subject to one stress which is Y, and if X is the only strength that is subject to k components of stress Y.

In this paper, the reliability Bayesian analysis to estimate the unknown shape parameters (α and θ) for X and Y under three loss functions is organized as follows:
In Section 2, we introduce the system reliability formulation. In Section 3, we consider the three methods for estimating reliabilities R_1 and R_2. In Section 4, we compare the estimators of R by Monte Carlo simulations. Finally, the results conclusions are given in Section 5.

2 The system reliability formulation

The formulation of two stress-strength model reliabilities of GP distribution can be obtained as follows: Let the strength random variables sample $(x_1, \ldots, x_k)$ be independent identically distributed with parameters $(\alpha_1 = \alpha_2 = \ldots = \alpha_k = \alpha; \lambda)$ and independent from stress (Y) random variable with $GP(\theta, \lambda)$. The reliability for k components exposed to one stress from the Generalized Pareto distribution can be expressed as:

$$\begin{aligned} R_1 &= P\left(y < \max\left(x_1, \ldots,\ x_k\right)\right) \\ &= 1 - \int_y \prod_{i=1}^{k} F_{x_i}\left(y\right) f\left(y\right) dy \\ &= 1 - \int_y \left(F_x\left(y\right)\right)^k f\left(y\right) dy \end{aligned} \tag{2.3}$$

by substituting in Eq. 2.3 as:

$$\begin{aligned} &= 1 - \int_y \left(1 - \left(1 + \lambda y\right)^{-\alpha}\right)^k \theta\lambda \left(1 + \lambda y\right)^{-(\theta+1)} dy \\ &= \ 1 - \int_y \sum_{j=0}^{k} C_j^k \left(-1\right)^j \left(1 + \lambda y\right)^{-j\alpha} \theta\lambda \left(1 + \lambda y\right)^{-(\theta+1)} dy \\ &= 1 - \sum_{j=0}^{k} C_j^k \left(-1\right)^j \theta \int_y \lambda \left(1 + \lambda y\right)^{-(j\alpha+\theta+1)} dy \end{aligned}$$

Then,

$$R_1 = 1 - \sum_{j=0}^{k} C_j^k \left(-1\right)^j \frac{\theta}{j\alpha + \theta} \tag{2.4}$$

Next, we consider the reliability for a component strength x with $GP(\alpha, \lambda)$ $(y_1, \ldots,\ y_k)$ stresses from GP with $(\theta_1 = \theta_2 = \ldots = \theta_k = \theta; \lambda)$ as:

$$\begin{aligned} R_2 &= P\left(\max\left(y_1, \ldots,\ y_k\right) < x\right) \\ &= \int_x \prod_{i=1}^{k} F_{y_i}\left(x\right) f\left(x\right) dx \\ &= \int_x \left(F_y\left(x\right)\right)^k f\left(x\right) dx \end{aligned}$$

$$= \int_x \left(1 - (1+\lambda x)^{-\theta}\right)^k \alpha\lambda\,(1+\lambda x)^{-(\alpha+1)}\,dx$$

$$= \int_x \sum_{j=0}^{k} C_j^k\,(-1)^j\,(1+\lambda x)^{-j\theta}\alpha\lambda\,(1+\lambda x)^{-(\alpha+1)}\,dx$$

$$= \sum_{j=0}^{k} C_j^k\,(-1)^j\alpha \int_x \lambda\,(1+\lambda x)^{-(j\theta+\alpha+1)}dx$$

Then,

$$R_2 = \sum_{j=0}^{k} C_j^k\,(-1)^j \frac{\alpha}{\alpha + j\theta} \tag{2.5}$$

3 Estimation of reliability

In this section, we discuss three estimators of the two unknown parameters α, θ and the two reliabilities R_1 and R_2 with generalized Pareto distribution under the doubly Type II censored data.

3.1 The likelihood function

Assuming that $X = x_{(1)}, x_{(2)}, \ldots, x_{(r)}, x_{(r+1)}, \ldots, x_{(s)}, x_{(s+1)}, \ldots, x_{(n)}$ from $X_i \sim GP(\alpha,\lambda)$ and $Y = y_{(1)}, y_{(2)}, \ldots, y_{(r)}, \ldots, y_{(s)}, \ldots, y_{(n)}$ from $Y_i \sim GP(\theta,\lambda)$; be two independent random variables from a GP distribution of a sample of size n. The statistical analysis contains the application of only the ordered remaining observations in the sample; that is, $x = x_{(r)}, x_{(r+1)}, \ldots, x_{(s)}$; the doubly censored data pulled from the sample with cdf and pdf as given in 1.1 and 1.2 equations, respectively. Then the likelihood function can be written as [8, 9]:
$L\,(x_i;\ \alpha,\lambda) = \prod_{i=r}^{s} f(x_i;\alpha,\lambda)$

$$= \frac{n!}{(r-1)!(n-s)!} \prod_{i=r}^{s} f(x_i)\left(F(x_{(r)})\right)^{r-1}\left(1 - F(x_{(s)})\right)^{n-s},$$

where $\prod_{i=r}^{s} f(x_i) = \alpha^{s-r+1}\lambda^{s-r+1}e^{-(\alpha+1)\sum_{i=r}^{s} ln_{(1+\lambda x_i)}}$
$\left(F(x_{(r)})\right)^{r-1} = \left(1 - e^{-\alpha ln\left(1+\lambda x_{(r)}\right)}\right)^{r-1}$
We can write:

$\left(F(x_{(r)})\right)^{r-1} = \sum_{j=0}^{r-1} C_j^{r-1} (-1)^j \, e^{-\alpha_j ln\left(1+\lambda x_{(r)}\right)}$, and

$$\left(1 - F(x_{(s)})\right)^{n-s} = \left(e^{-\alpha ln\left(1+\lambda x_{(s)}\right)}\right)^{n-s} = e^{-\alpha(n-s)ln\left(1+\lambda x_{(s)}\right)}$$

Then $L\left(x_i;\ \alpha, \lambda\right) = \frac{n!}{(r-1)!(n-s)!} \alpha^{s-r+1} \lambda^{s-r+1} e^{-(\alpha+1)\sum_{i=r}^{s} ln\left(1+\lambda x_i\right)} e^{-\alpha(n-s)\ln\left(1+\lambda x_{(s)}\right)}$

$$\sum_{j=0}^{r-1} C_j^{r-1} (-1)^j \, e^{-\alpha_j \ln\left(1+\lambda x_{(r)}\right)} \tag{3.6}$$

The gamma distribution is used as a prior distribution due to its wide importance in Bayesian analysis. Let α, θ be dependent and unknown parameters and consider the reliability of these parameters as random variables under gamma with common parameters (a, b).

$$g\left(\alpha\right) = \frac{b^a}{\Gamma a} \alpha^{a-1} e^{-b\alpha} \tag{3.7}$$

And,

$$g\left(\theta\right) = \frac{b^a}{\Gamma a} \theta^{a-1} e^{-b\theta} \tag{3.8}$$

for $\alpha, \theta > 0$ and $a, b > 0$. Using the likelihood function in Eq. 3.6 and the gamma prior density Eq. 3.7, we have:

$$L\left(x_i;\ \alpha, \lambda\right)\ g\left(\alpha\right) = \frac{n!}{(r-1)!(n-s)!} \alpha^{s-r+1} \lambda^{s-r+1} e^{-(\alpha+1)\sum_{i=1}^{n} \ln\left(1+\lambda x_i\right)}$$
$$\frac{b^a}{\Gamma \mathrm{a}} \alpha^{a-1} e^{-b\alpha} e^{-\alpha(n-s)\ln\left(1+\lambda x_{(s)}\right)} \sum_{j=0}^{r-1} C_j^{r-1} (-1)^j \, e^{-\alpha_j \ln\left(1+\lambda x_{(r)}\right)} \tag{3.9}$$

$$L\left(x_i;\ \alpha, \lambda\right) g\left(\alpha\right) = \frac{n!\ b^a \lambda^{s-r+1}\ e^{-\sum_{i=1}^{n} \ln\left(1+\lambda x_i\right)}}{(r-1)!(n-s)!\Gamma \mathrm{a}} \alpha^{s-r+a}$$
$$\sum_{j=0}^{r-1} C_j^{r-1} (-1)^j \ \ e^{-\alpha\left(\sum_{i=1}^{n} \ln\left(1+\lambda x_i\right) + (n-s) ln\left(1+\lambda x_{(s)}\right) + j \ln\left(1+\lambda x_{(r)}\right) + b\right)} \tag{3.10}$$

Rewrite Eq. 3.10 and assume that: $Q = \frac{n!\ b^a \lambda^{s-r+1}\ e^{-\sum_{i=1}^{n} \ln\left(1+\lambda x_i\right)}}{(r-1)!(n-s)!\Gamma \mathrm{a}}$;
and $A_j = \sum_{i=1}^{n} \ln\left(1+\lambda x_i\right) + (n-s)\ln\left(1+\lambda x_{(s)}\right) + j\ln\left(1+\lambda x_{(r)}\right) + b$

Then $\int L\left(x_i;\alpha,\lambda\right)g\left(\alpha\right)d\alpha = Q\sum_{j=0}^{r-1} C_j^{r-1}\left(-1\right)^j \int_\alpha \alpha^{s-r+a}e^{-A_j\alpha}d\alpha$ Finally, we obtain the likelihood function $g\left(\alpha\right)$ as the prior function for the shape parameter α from $x_i \sim GP$

$$\int L\left(x_i;\ \alpha,\lambda\right)\ g\left(\alpha\right)d\alpha = Q\sum_{j=0}^{r-1} C_j^{r-1}\left(-1\right)^j \frac{\Gamma(s-r+a+1)}{A_j^{s-r+a+1}} \tag{3.11}$$

Similarly, we obtain the function for the shape parameter θ from $y_i \sim GP$, as:

$$\int L\left(y_i;\ \theta,\lambda\right)\ g\left(\theta\right)d\theta = K\sum_{j=0}^{r-1} C_j^{r-1}\left(-1\right)^j \frac{\Gamma(s-r+a+1)}{B_j^{s-r+a+1}}, \tag{3.12}$$

where $K = \frac{n!\ b^a\lambda^{s-r+1}\ e^{-\sum_{i=1}^n ln(1+\lambda y_i)}}{(r-1)!(n-s)!\Gamma a}$;
and $B_j = \sum_{i=1}^n \ln\left(1+\lambda y_i\right) + \left(n-s\right) ln\big(1+\lambda y_{(s)}\big) + jln\big(1+\lambda y_{(r)}\big) + b$

3.2 The posterior distribution

The objective of this section is to find Bayesian estimators of the shape parameters α, θ by using the three various loss functions [10]. The posterior function for $x_i \sim GP(\alpha,\lambda)$ and $y_i \sim GP(\theta,\lambda)$ are:

$$\begin{aligned} p\left(\frac{\alpha}{\underline{x}}\right) &= \frac{l(\frac{\alpha}{\underline{x}})g\left(\alpha\right)}{\int_0^\infty l(\frac{\alpha}{\underline{x}})g\left(\alpha\right)d\alpha} \\ &= \frac{Q\sum_{j=0}^{r-1} C_j^{r-1}\left(-1\right)^j \alpha^{s-r+a}e^{-A_j\alpha}}{Q\sum_{j=0}^{r-1} C_j^{r-1}\left(-1\right)^j\ \frac{\Gamma(s-r+a+1)}{A_j^{s-r+a+1}}} \end{aligned} \tag{3.13}$$

In addition,

$$\begin{aligned} p\left(\frac{\theta}{\underline{y}}\right) &= \frac{l\left(\frac{\theta}{\underline{y}}\right)g\left(\theta\right)}{\int_0^\infty l\left(\frac{\theta}{\underline{x}}\right)g\left(\theta\right)d\theta} \\ &= \frac{K\sum_{j=0}^{r-1} C_j^{r-1}\left(-1\right)^j \alpha^{s-r+a}e^{-B_j\alpha}}{K\sum_{j=0}^{r-1} C_j^{r-1}\left(-1\right)^j\ \frac{\Gamma(s-r+a+1)}{B_j^{s-r+a+1}}} \end{aligned} \tag{3.14}$$

3.2.1 Squared error loss function SELF

The derivation of Bayes estimator under SELF is given by [11]:

$$\hat{\alpha}_s = E\left(\frac{\alpha}{\underline{x}}\right) = \int_\alpha \alpha p\left(\frac{\alpha}{\underline{x}}\right) d\alpha$$
$$= \frac{\sum_{j=0}^{r-1} C_j^{r-1} (-1)^j}{\sum_{j=0}^{r-1} C_j^{r-1} (-1)^j \frac{\Gamma(s-r+a+1)}{A_j^{s-r+a+1}}} \int_\alpha \alpha^{s-r+a+1} e^{-A_j \alpha} d\alpha.$$

Let $Z_j = \sum_{j=0}^{r-1} C_j^{r-1} (-1)^j \frac{\Gamma(s-r+a+1)}{A_j^{s-r+a+1}}$. Then $\hat{\alpha}_s = \frac{1}{Z_j} \sum_{j=0}^{r-1} C_j^{r-1} (-1)^j$ $\frac{\Gamma(s-r+a+2)}{A_j^{s-r+a+2}}$ Also, $\hat{\theta}_s = \frac{1}{W_j} \sum_{j=0}^{r-1} C_j^{r-1} (-1)^j \frac{\Gamma(s-r+a+2)}{B_j^{s-r+a+2}}$; in which $W_j = \sum_{j=0}^{r-1} C_j^{r-1}$ $(-1)^j \frac{\Gamma(s-r+a+1)}{B_j^{s-r+a+1}}$ Thus the reliability estimation function by Eqs. 2.4 and 2.5 is given by:

$$\hat{R}_{1s} = 1 - \sum_{j=0}^{k} C_j^k (-1)^j \frac{\hat{\theta}_s}{j\hat{\alpha}_s + \hat{\theta}_s} \tag{3.15}$$

And,

$$\hat{R}_{2s} = \sum_{j=0}^{k} C_j^k (-1)^j \frac{\hat{\alpha}_s}{\hat{\alpha}_s + j\hat{\theta}_s} \tag{3.16}$$

3.2.2 De Groot loss function DLF

We can also use the Bayes estimator DeGroot Loss Functiont defined as [12]: $\hat{\alpha}_D = \frac{E\left(\frac{\alpha^2}{\underline{x}}\right)}{E\left(\frac{\alpha}{\underline{x}}\right)}$ and $\hat{\theta}_D = \frac{E\left(\frac{\theta^2}{\underline{y}}\right)}{E\left(\frac{\theta}{\underline{y}}\right)}$. In which, $E\left(\frac{\alpha^2}{\underline{x}}\right) = \frac{1}{Z_j} \sum_{j=0}^{r-1} C_j^{r-1} (-1)^j$ $\int_\alpha \alpha^{s-r+a+2} e^{-A_j \alpha} d\alpha$

$$E\left(\frac{\alpha^2}{\underline{x}}\right) = \frac{1}{Z_j} \sum_{j=0}^{r-1} C_j^{r-1} (-1)^j \frac{\Gamma(s-r+a+3)}{A_j^{s-r+a+3}} \tag{3.17}$$

Also, $E\left(\frac{\theta^2}{\underline{y}}\right) = \frac{1}{W_j} \sum_{j=0}^{r-1} C_j^{r-1} (-1)^j \int_\theta \alpha^{s-r+a+3} e^{-B_j \alpha} d\theta$

$$E\left(\frac{\theta^2}{\underline{y}}\right) = \frac{1}{W_j} \sum_{j=0}^{r-1} C_j^{r-1} (-1)^j \frac{\Gamma(s-r+a+3)}{B_j^{s-r+a+3}} \tag{3.18}$$

Using Eqs. 3.17 and 3.18 for $\hat{\alpha}_D$ and $\hat{\theta}_D$, the reliability estimation function by Eqs. 2.4 and 2.5 is given as:

$$\hat{R}_{1D} = 1 - \sum_{j=0}^{k} C_j^k (-1)^j \frac{\hat{\theta}_D}{j\hat{\alpha}_D + \hat{\theta}_D} \tag{3.19}$$

and

$$\hat{R}_{2D} = \sum_{j=0}^{k} C_j^k (-1)^j \frac{\hat{\alpha}_D}{\hat{\alpha}_D + j\hat{\theta}_D} \tag{3.20}$$

3.2.3 Precautionary loss function

Finally, using the Bayes estimator of the Precautionary Loss Function from Eqs. 3.17 and 3.18 [8]: $\hat{\alpha}_p = \left(E\left(\frac{\alpha^2}{\underline{x}}\right)\right)^{\frac{1}{2}}$ and $\hat{\theta}_p = \left(E\left(\frac{\theta^2}{\underline{y}}\right)\right)^{\frac{1}{2}}$

$$\hat{\alpha}_p = \left(\frac{1}{Z_j} \sum_{j=0}^{r-1} C_j^{r-1} (-1)^j \frac{\Gamma(s-r+a+3)}{A_j^{s-r+a+3}}\right)^{\frac{1}{2}} \tag{3.21}$$

and

$$\hat{\theta}_p = \left(\frac{1}{W_j} \sum_{j=0}^{r-1} C_j^{r-1} (-1)^j \frac{\Gamma(s-r+a+3)}{B_j^{s-r+a+3}}\right)^{\frac{1}{2}} \tag{3.22}$$

Then, by Eqs. 3.21 and 3.22, the reliability estimation function is given as:

$$\hat{R}_{1P} = 1 - \sum_{j=0}^{k} C_j^k (-1)^j \frac{\hat{\theta}_P}{j\hat{\alpha}_P + \hat{\theta}_P} \tag{3.23}$$

and

$$\hat{R}_{2P} = \sum_{j=0}^{k} C_j^k (-1)^j \frac{\hat{\alpha}_P}{\hat{\alpha}_P + j\hat{\theta}_P} \tag{3.24}$$

In order to study the behavior of these estimators in Eqs. 3.15, 3.16, 3.19, 3.20, 3.23 and 3.24 for the two reliabilities and to identify the best achievement among them, a simulation study was conducted to compare among these estimators.

4 Simulation Study

A simulation study was performed by using MATLAB 2018 to compare the performance of the discussed estimators for the best reliability estimate of the stress-strength based on samples generated from the generalized Pareto distribution using doubly Type II censored data. The results are based on Monte Carlo simulation of the different sample sizes $n = 15, 30, 60$ and applied for different values of $[(r, s) = (6, 10), (11, 20)]$ with different parameters values of θ, α. To generate the different values of the random variable X from the cumulative distribution function and using the inverse technique of distribution function: $F = 1-(1+\lambda x)^{-\alpha}$; $\left(1-F = (1+\lambda x)^{-\alpha}\right)^{\frac{-1}{\alpha}}$; $1+\lambda x = (1-F)^{\frac{-1}{\alpha}}$; Then, $x = \frac{1}{\lambda}\left((1-F)^{\frac{-1}{\alpha}} - 1\right)$ The results are recorded in Tables 1-6 with each table containing the estimates of the reliability for GP distribution under three various loss functions (Squared Error, De-Groot and Precautionary) with the efficiency of the estimators MSE for the different sample sizes and parameters values. From Table 1, it is clear that the achievement preference differed between the Squared Error and Precautionary smallest sample size $n = 15$ and different parameters values for reliabilities R_1 and R_2, but the preference was the Precautionary loss function for both dependencies in the Tables 2-6, and the Squared Error loss function in the Table 3. Noting that there is no best performance under the effect of De-Groot loss function under those different experimental values of the distribution parameters, sample sizes and component numbers.

Table 1: Experiment 1 results

Exp.1 $\theta = 0.4,\ \alpha = 0.7, \lambda = 0.9,\ n = 15,\ r = 6, s = 10, k = 3$				
Real reliabilities values R_1=0.877511961722488 R_2=0.584242424242424				
	S	D	P	Best
R_1 mean	0.861457116	0.911385290	0.861457585	P
MSE	0.000257758	0.001147402	0.000257743	
R_2 Mean	0.590430699	0.445463725	0.590431087	S
MSE	0.000038295	0.019259527	0.000038300	

Table 2: Experiment 2 results

Exp.2 $\theta = 0.3,\ \alpha = 0.5, \lambda = 0.9,\ n = 15,\ r = 6, s = 10, k = 3$				
Real reliabilities values R_1=0.868506493506494 R_2=0.599358974358974				
	S	D	P	Best
R_1 mean	0.857072240	0.907952314	0.857072279	P
MSE	0.000130742	0.001555973	0.000130741	
R_2 Mean	0.599073415	0.459849115	0.599074290	P
MSE	0.000000082	0.019463001	0.000000081	

Table 3: Experiment 3 results

Exp.3 $\theta = 0.4,\ \alpha = 0.7, \lambda = 0.9,\ n = 30,\ r = 11, s = 20, k = 3$				
Real reliabilities values R_1=0.877511961722488 R_2=0.584242424242424				
	S	D	P	Best
R_1 mean	0.86960433	0.93131922	0.86957347	S
MSE	0.00006253	0.00289522	0.00006302	
R_2 Mean	0.58689741	0.43172121	0.58696388	S
MSE	0.00000705	0.02326272	0.00000741	

Table 4: Experiment 4 results

Exp.4 $\theta = 0.3,\ \alpha = 0.5, \lambda = 0.9,\ n = 30,\ r = 11, s = 20, k = 3$				
Real reliabilities values R_1=0.868506493506494 R_2=0.599358974358974				
	S	D	P	Best
R_1 mean	0.85999925	0.92043687	0.86001709	P
MSE	0.00007237	0.00269676	0.00007207	
R_2 mean	0.60288125	0.46039319	0.60287730	P
MSE	0.00001241	0.01931149	0.00001238	

Table 5: Experiment 5 results

Exp.5 $\theta = 0.4,\ \alpha = 0.7, \lambda = 0.9,\ n = 60,\ r = 30, s = 45, k = 3$				
Real reliabilities values R_1=0.877511961722488 R_2=0.584242424242424				
	S	D	P	Best
R_1 mean	0.84549319	0.91686782	0.86433051	P
MSE	0.0010252	0.00154888	0.00017375	
R_2 Mean	0.58031448	0.43468788	0.58470295	P
MSE	1.54E-05	0.02236656	2.12E-07	

Table 6: Experiment 6 results

Exp.6 $\theta = 0.3,\ \alpha = 0.5, \lambda = 0.9,\ n = 30,\ r = 11, s = 20, k = 3$				
Real reliabilities values R_1=0.868506493506494 R_2=0.599358974358974				
	S	D	P	Best
R_1 mean	0.85999925	0.92043687	0.86001709	P
MSE	0.00007237	0.00269676	0.00007207	
R_2 mean	0.60288125	0.46039319	0.60287730	P
MSE	0.00001241	0.01931149	0.00001238	

5 Conclusions

We compared three estimators of reliability for stress–strength Generalized Pareto Bayesian model, by means of squared error MSE criteria, taking different sample sizes. We performed Monte Carlo simulations to compare the different proposed methods. We noticed that the best achievement for the two Reliabilities, whose behavior were studied under the various experiments mentioned above, was the Precautionary loss function under the doubly type II censored data.

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