International Journal of Mathematics and
Computer Science, **19**(2024), no. 4, 1171–1175

Analysis of D-Wave topologies with classical graph metrics

Csaba Biró

Faculty of Informatics
Eszterházy Károly Catholic University and Eötvös Lóránd University
Eger and Budapest, Hungary

biro.csaba@uni-eszterhazy.hu, birocsaba@staff.elte.hu

(Received March 30, 2024, Accepted May 1, 2024,
Published June 1, 2024)

Abstract

In this paper, we focus on graph-based analysis of the topology of D-Wave quantum computers. The Pegasus, Chimera and Zephyr topologies generated with different parameters are examined using classical based graph metrics. Our main goal is to use metrics to highlight the main features and limitations of these topologies. The secondary goal is that the results contribute to the further development of more efficient quantum processors.

1 Introduction

In this article, we focus on D-Wave processor topologies. The processors of D-Wave work with quantum annealing that are used to solve optimization problems. The topology of D-Wave's quantum annealing processors is based on a Chimera or a Pegazus or a Zephyr graph [2, 3, 4]. These topologies are optimized for certain types of optimization problems, such as the Ising model and the quadratic unconstrained binary optimization problem. D-Wave's processors are not universal and may not be suitable for other types of quantum algorithms that require different types of connectivity or topology [1, 6, 8].

Key words and phrases: D-wave, Chimera, Pegasus, Zephyr, classical graph metrics.
AMS (MOS) Subject Classifications: 68R10, 05C90, 05C75, 05C90, 81Q35.
ISSN 1814-0432, 2024, http://ijmcs.future-in-tech.net

2 Main results

Chimera two-dimensional lattice graph $\mathcal{C}_{M,N,L}$ is an $\mathcal{M}$ x $\mathcal{N}$ grid of Chimera tiles implementing the topology of the D-Wave 2000Q systems. The Chimera titles $\mathcal{K}_{L,L}$ are complete bipartite graphs. Chimera graph contains a particularly nice clique minor and so triangle embedding is uniform and near-optimal [2, 5]. The following configurations were used on Chimera-based D-Wave processors: $\mathcal{C}_{l\cdot n,l\cdot n,l}$ where $n \in \mathbf{N}^+$ and $l = 4$. The Chimera graphs were examined under different configurations from $\mathcal{C}_{2,2,4}$ to $\mathcal{C}_{16,16,4}$ with $\mathcal{L}$ values of 4 and 8.

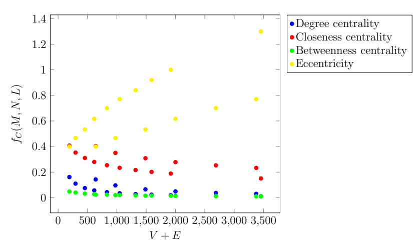

Figure 1: The average values of degree, closeness, betweenness centrality and eccentricity/10 as a function of $V + E$ for different Chimera graphs.

Let $f_C(M, N, L)$ be a function for representing a Chimera graph $G(V, E)$ as a function of $V + E$. The value set of f_C is given by the average values of one of the previously defined classical or k-hop based metrics. The parameters N and M primarily define the characteristic of the function, while L the offset of f_C on the y axis. Figure 1 shows f_C functions of classical metrics as a function of $V + E$ for different Chimera graphs.

The Pegasus topology is a variation of the Chimera architecture. In case of this topology each tile has eight qubits arranged in a square, and each qubit is connected to its four nearest neighbors by couplers. The couplers are essentially the connections between qubits. One of the important features of the Pegasus topology is that it also has longer-range couplers that connect qubits between adjacent tiles. It is suitable for performing more complex calculations than the Chimera-based processors, because it allows for more complex interactions between qubits. The Pegasus topology is designed to provide a high level of connectivity and flexibility to support a wider range of quantum algorithms than earliers. By definition, has some disconnected components, the $\mathcal{P}_0$ contains $8(M-1)$ qubits. The size of the main processor fabric is $24M(M-1) - 8(M-1)$, the size of the full disconnected graph is $24M(M-1)$. A Pegasus graph that retains the basic characteristics of Chimera can also be created in this case and the size of the graph is $24(M-1)^2$. The Pegasus graphs were examined under different configurations from $\mathcal{P}_2$ to $\mathcal{P}_8$. Let $f_P(M)$ be a function for representing a Pegasus graph $G(V,E)$ as a function of $V+E$.

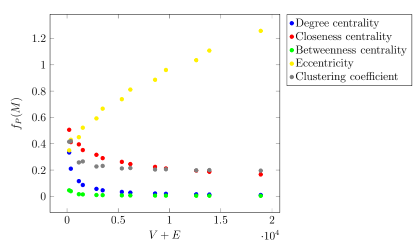

Figure 2: The average values of clustering coefficient, degree, betweenness, closeness centrality and eccentricity/10 as a function of $V+E$ for different Pegasus graph.

Zephyr provides the topology of D-Wave's latest generation processors. In Zephyr, as in Pegasus and Chimera, qubits are oriented either vertically or horizontally. The Zephyr topology includes the basic coupler types of both Chimera and Pegasus, with a total of two odd couplers, two external couplers, and sixteen internal couplers. In this topology, the nominal length and degree of qubits are respectively, 16 and 20. The two basic parameters of the Zephyr $\mathcal{Z}_{M,T}$ are M and T. The maximum degree of this graph is $4T+4$, and the number of nodes is $4TM(2+1)$ [4, 6].

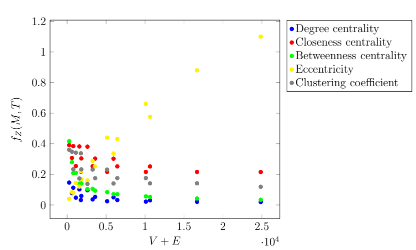

Figure 3: Clustering coefficient, degree, closeness, 10*betweenness centrality and eccentricity/100 values as a function of $V+E$ for different Zephyr graphs.

The Zephyr index of a vertex in a Zephyr lattice depends on multiple parameters: u,w,k,j,z, respectively orientation, perpendicular block offset, qubit index, shift identifier, parallel tile offset. The Zephyr graphs were examined under different configurations from $\mathcal{Z}_{2,1}$ to $\mathcal{Z}_{5,5}$. Let $f_Z(M,T)$ be a function for representing a Zephyr graph $G(V,E)$ as a function of $V+E$. The value set of f_Z is given by the average values of one of the previously defined classical based metrics.

3 Conclusion and future work

We examined the characteristics shown by classical metrics. The average values of the classical metrics are shown in Figures 1-3. In the case of Chimera, as previously mentioned, the case separation can be seen depending on the value of L. Figure 1 shows that in the case of $L = 8$ (bottom line of dots) the average eccentricity values are smaller than in the case of $L = 4$ (top line of dots). This is natural since L defines the size of the shore within each Chimera tile. Thus in the case of a larger L the average length of the paths are smaller. In the case of the Pegasus graph, the examined parameters do not affect the nature of the function in the average eccentricity values (Figure 2). In the case of the Zephyr graph, the averages of the eccentricity values show an interesting feature (Figure 3). We conclude that this metric is independent of the value of $V+E$ and only depends on the value of M. The average values of degree and closeness centrality and clustering coefficient show (Figures 1 and 2) the same characters in the Chimera and Pegasus graphs as in the case of eccentricity (in the case of Chimera, the strong dependence on the parameter L is clearly visible while in the case of the Pegasus graph it is independent of the parameters), naturally the nature of the curves is different, since the values of the metrics decrease as a function of $V+E$. In the case of the Zephyr graph, a strong dependence on the M parameter can also be observed. The average values of betweenness centrality is the classical metric that shows (see Figures 1-3 the same character for all three graphs. In the case of Chimera, a weaker dependence on the value of L is visible but in the other cases it can be said that it is independent of the parameters; i.e., the parameters do not impact the nature of the curve. In this paper, we focused on graph-based analysis of the topology of D-Wave quantum computers. We analyzed Chimera, Pegasus and Zephyr graphs with different configurations. In general, we worked the average values of the metrics interpreted on the graphs as we were interested in the characteristics shown by the metrics. Our main goal was to highlight the main features and limitations of topologies through metrics. In the future, our goal is to examine Zephyr and newer topologies more deeply, focusing on cliques and bicliques, heuristic 2D and 3D lattice embeddings, and heuristic optimization.

Acknowledgment. This research was supported by the Ministry of Culture and Innovation and the National Research, Development and Innovation Office within the Quantum Information National Laboratory of Hungary (Grant No. 2022-2.1.1-NL-2022-00004).

References

[1] C. Biró, G. Kusper, Some k-hop Based Graph Metrics and Node Ranking in Wireless Sensor Networks, *Annales Mathematicae et Informaticae*, **50,** (2019), 19–37.

[2] Boothby, A.D. King, A. Roy, Fast clique minor generation in Chimera qubit connectivity graphs, *Quantum Information Processing*, **15,** (2016), 495–508.

[3] K. Boothby, P. Bunyk, J. Raymond, A. Roy, Next-generation topology of d-wave quantum processors, *arXiv preprint arXiv:2003.00133*, (2020).

[4] K. Boothby, A. King, J. Raymond, Zephyr Topology of D-Wave Quantum Processors, *D-wave technical report series*, (2021).

[5] V. Choi, Minor-embedding in adiabatic quantum computation II. Minor-universal graph design, *Quantum Inf. Process*, **10,** no. 3, (2011), 343–353.

[6] T. Huang, Y. Zhu, R.S.M. Goh, R. T. Luo, When quantum annealing meets multitasking: Potentials, challenges and opportunities, *Array*, (2023), 100282.

[7] V. Lahtinen, J. Pachos, A short introduction to topological quantum computation, *SciPost Physics*, **3,** no. 3, (2017), 021.

[8] E. Pelofske, Comparing Three Generations of D-Wave Quantum Annealers for Minor Embedded Combinatorial Optimization Problems, *arXiv preprint arXiv:2301.03009*, (2023).