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A class of Lotka-Volterra operators and its application (action in the S^3 simplex)

Sirojiddin Masharipov[1], **Bakhodir Kholboev**[2], **Dmitriy Kim**[3]

[1]Tashkent University of Information Technologies named after Muhammad al-Khwarizmi
Tashkent, Uzbekistan

[2]G.V. Plekhanov Russian University of Economics Tashkent branch
Tashkent, Uzbekistan

[3]National University of Uzbekistan named after Mirzo Ulugbek
Tashkent, Uzbekistan

email: bakhodir.kholboev@gmail.com

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Abstract

The work is devoted to behaviors and some applied fields in a discrete dynamical system in a S^3 simplex. Moreover, we study the motion of the trajectory using simulation instead of four-dimensional space drawing in a three-dimensional space.

1 Introduction

The theory of quadratic operators was first formulated by Bernstein [1]. Lyubich [2] changed the evolutionary operator so that instead of coefficients ± 1 in the equation $x_1 = 1 - x_2 - x_3$ there will be coefficients a_{ij}. When studying dynamics by using Jacobian calculations, it is possible to determine whether

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the trajectory is attractive or repulsive. From these various listed Lotka-Volterra operators, it can be seen that P and Q consist of an infinite point and that they certainly consist of a fixed point [3].

Theorem 1.1. *If x^0 is the inner point of the simplex $S^{m-1} = \{x \in R^m : \sum_{i=1}^{m} x_i = 1,\ x_i \geq 0\}$, then $\alpha(x^0) \in P$; $P = \{x \in S^{m-1} : Ax \geq 0\} \neq \emptyset$ and $\omega(x^0) \in Q$; $Q = \{x \in S^{m-1} : Ax \leq 0\} \neq \emptyset$ with both positive and negative trajectories converging*

2 Main Results

Let $A = \begin{pmatrix} 0 & -a & 0 & -d \\ a & 0 & c & -f \\ 0 & -c & 0 & e \\ d & f & -e & 0 \end{pmatrix}$, with $0 < a, b, c, d \leq 1$, which corresponds to partially-oriented graph in Figure 1*. From $P = \{x \in S^3 : Ax \geq 0\}$ and $Q = \{x \in S^3 : Ax \leq 0\}$, we get $P = \{(\frac{e+d(1-\lambda)}{d+e}, 0, \frac{\lambda d}{d+e}, 0),\ 0 \leq \lambda \leq 1\}$, $Q = \{(0, \frac{e}{f+c+e}, \frac{f}{f+c+e}, \frac{c}{f+c+e})\}$. In this case, $V : S^3 \to S^3$ defined by the A matrix has the form

$$\begin{aligned} x_1' &= x_1 \cdot (1 - ax_2 - dx_4) \\ x_2' &= x_2 \cdot (1 + ax_1 + cx_3 - fx_4) \\ x_3' &= x_3 \cdot (1 - cx_2 + ex_4) \\ x_4' &= x_4 \cdot (1 + dx_1 + fx_2 - ex_3) \end{aligned} \tag{1}$$

Calculating the Jacobian of fixed points in the simplex, we get
$J(V(x)) = (1-\lambda)^2 \cdot (1 - ax_1 - cx_3 - \lambda) \cdot (1 - dx_1 + ex_3 - \lambda)$.
Hence the Jacobian eigenvalues are:
$\lambda_1 = \lambda_2 = 1;\ \lambda_3 = 1 - ax_1 - cx_3;\ \lambda_4 = 1 - dx_1 + ex_3$.
Moreover, $\lambda_3 = 1 - ax_1 - cx_3 < 1$. If $x_1 \leq \frac{e}{e+d}$, then $\lambda_4 = 1 - dx_1 + ex_3 < 1$ for all points from Γ_{13}. Therefore, $\omega(x^0) \in Q$. Similarly, $\alpha(x^0) \in P$. (Fig.1*)

3 Simulation

It is necessary to represent the system (1) as a system of partial derivatives. To do this, we slightly modify the system $x_k' = x_{k+1} - x_k = \sum_{i=1}^{m} a_{ki} x_i x_k,\ k =$

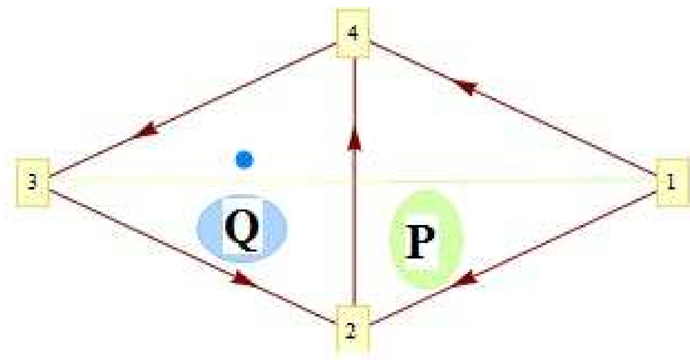
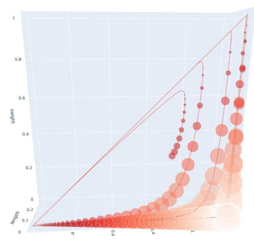


Figure 1*

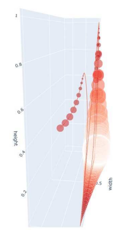
$\overline{1,m}$. Obviously, the differentiation variable will be the number of steps; that is, $dt = k + 1 - k = 1$ since the distance between steps is always equal to one. Let's introduce this system into the Jupyter Notebook program in the following form:

$$
\begin{gathered}
x_1' = ax_1x_2 + dx_1x_4 \\
x_2' = ax_1x_2 - cx_2x_3 + fx_2x_4 \\
x_3' = cx_2x_3 + ex_3x_4 \\
x_4' = -dx_1x_4 - fx_2x_4 - ex_3x_4
\end{gathered}
$$

Having an array with data satisfying the conditions $x_m' + y_m' + z_m' + k_m' = 1$ and $x_m', y_m', z_m', k_m' \geq 0$, we can build a 3D-diagram taking the first three variables as length, width, height. Let us take the fourth variable as time which will determine the radius and depth of color of the spheres, the center of which will be determined by the first three coordinates. We obtain that a sequence of spheres with a nonlinearly increasing (decreasing) radius. This clearly demonstrates action of the graph in Figure 1*. That is, the movement from top 1 to the graph 3,2,4 can be seen as a sequence of spheres which converges to a point Q in the graph 3,2,4. Point Q is located where spheres have almost white color.



a. With time serving as parameter of color depth and radius of spheres



b. With time serving only as parameter of color depth

Figure 1: Simulation of System (1) for Figure 1*

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