International Journal of Mathematics and
Computer Science, **19**(2024), no. 4, 1281–1285

$\mu_{mn}S_p$-Open Sets in Bigeneralized Topological Spaces

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(Received April 25, 2024, Accepted May 26, 2024,
Published June 1, 2024)

Abstract

In this paper, we introduce and characterize the notion of $\mu_{mn}S_p$-Open Sets, $\mu_{mn}S_p$-interior, and $\mu_{mn}S_p$-closure of a set in Bigeneralized Topological Spaces.

1 Introduction

In 2002, Császár introduced the concept of generalized topology [5]. Several counterparts of existing concepts in topology were defined including the μ-semiopen sets and μ-preopen sets.

Benjamin and Rara [4] introduced and characterizes the concepts of μS_p-open sets, μS_p-closed sets, μS_p-interior and μS_p-closure of a set in the generalized topological spaces. These concepts are generalized topology's counterpart of the S_p-open sets in [7].

Boonpok [3] introduced the concept of bigeneralized topological spaces. In this paper, we introduce and characterize the notions of $\mu_{mn}S_p$-Open Sets, $\mu_{mn}S_p$-interior, and $\mu_{mn}S_p$-closure of a set in Bigeneralized Topological Spaces.

Key words and phrases: $\mu_{mn}S_p$-Open sets, $\mu_{mn}S_p$-interior, and $\mu_{mn}S_p$-closure, Bigeneralized Topological Spaces
AMS (MOS) Subject Classifications: 54A05, 54A40, 54D40.
ISSN 1814-0432, 2024, http://ijmcs.future-in-tech.net

2 Preliminaries

Let X be a nonempty set. A subset μ of $P(X)$ is said to be a generalized topology (briefly GT) on X if $\varnothing \in \mu$ and the arbitrary union of elements of μ belongs to μ. If μ is a GT on X, then the pair (X, μ) is called a generalized topological space (briefly GT-space), and the elements of μ are called μ-open sets. The complement of a μ-open set is called a μ-closed set.

Throughout this paper, the space (X, μ_1, μ_2) (or simply X) mean a bigeneralized topological space (BGT-space) with no separation axioms unless otherwise stated. Let A be a subset of a bigeneralized topological spaces. The closure and the interior of A with respect to μ_m are denoted by $c_{\mu_m}(A)$ and $i_{\mu_m}(A)$, respectively, with $m = 1, 2$.

In 2019, Fathima et. al [2] introduced the following definition:
Let (X, μ_1, μ_2) be a bigeneralized topological space. Let A be a subset of X. Then A is said to μ_{mn}-semiopen if $A \subseteq c_{\mu_m}(i_{\mu_n}(A))$, where $m, n = 1, 2$ and $m \neq n$. The complement of a μ_{mn}-semiopen set is called a μ_{mn}-semiclosed set.

Moreover, in 2020, Rani et. al [1] introduced the notion of a μ_{mn}-preopen set as follows:
Let (X, μ_1, μ_2) be a bigeneralized topological space. Let A be a subset of X. Then A is said to μ_{mn}-preopen if $A \subseteq i_{\mu_m}(c_{\mu_n}(A))$, where $m, n = 1, 2$ and $m \neq n$. The complement of a μ_{mn}-preopen set is called a μ_{mn}-preclosed set.

3 $\mu_{mn}S_p$-Open Sets in the Bigeneralized Topological spaces

In this section, we introduce the notion of $\mu_{mn}S_p$-Open Sets in the Bigeneralized Topological spaces.

Definition 3.1. A subset A of a bigeneralized topological space X is called $\mu_{mn}S_p$*-open* if A is μ_n-semiopen and for every $x \in A$, there exists a μ_m-preclosed set F_x such that $x \in F_x \subseteq A$. The complement of a $\mu_{mn}S_p$-open set is called a $\mu_{mn}S_p$*-closed* set.

Remark 3.2. *Let* (X, μ_m, μ_n) *be a bigeneralized topological space. Then* A *is a* $\mu_{mn}S_p$*-open set in* X *if and only if* A *is* μ_n*-semiopen and* $A = \bigcup_{x \in A} F_x$*, where* F_x *is a* μ_m*-preclosed set.*

Remark 3.3. *The concepts of $\mu_m S_p$-open set or $\mu_n S_p$-open set and the $\mu_{mn}S_p$-open sets are independent notions.*

Remark 3.4. *The $\mu_{12}S_p$-open sets need not be $\mu_{21}S_p$-open. To see this, let $X = \{a, b, c, d\}$ with generalized topologies $\mu_1 = \{\varnothing, \{a, c\}, \{d\}, \{a, c, d\}\}$ and $\mu_2 = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}$. Then $\{a, c, d\}$ is a $\mu_{21}S_p$-open set but not $\mu_{12}S_p$-open.*

Theorem 3.5. *Let (X, μ_m, μ_n) be a bigeneralized topological space. Then A is a $\mu_{mn}S_p$-closed set in X if and only if A is μ_n-semiclosed and for every $x \notin A$, there exists a μ_m-preopen set U_x such that $A \subseteq U_x$.*

Proof.
Let A be a $\mu_{mn}S_p$-closed set in X. Then $X\backslash A$ is $\mu_{mn}S_p$-open. By Definition 3.1, $X\backslash A$ is μ_n-semiopen and for every $x \in X\backslash A$, there exists a μ_m-preclosed set F_x such that $x \in F_x \subseteq X\backslash A$. Hence A is μ_n-semiclosed and for every $x \notin A$, there exists a μ_m-preopen set $X\backslash F_x$ such that $A \subseteq X\backslash F_x$. Take $U_x = X\backslash F_x$. Thus the necessity of the theorem follows. The sufficiency is proved similarly. This completes the proof. □

Definition 3.6. The union of all the $\mu_{mn}S_p$-open sets of a BGT-space X contained in $A \subseteq X$ is called the $\mu_{mn}S_p$-*interior of* A, denoted by $\mu_{mn}S_p i_{\mu_{mn}}(A)$.

Remark 3.7. For any subset A of a BGT-space X, $\mu_{mn}S_p i_{\mu_{mn}}(A) \subseteq A$.

Definition 3.8. The intersection of all the $\mu_{mn}S_p$-closed sets of X containing A is called the $\mu_{mn}S_p$-*closure* of A, denoted by $\mu_{mn}S_p c_{\mu_{mn}}(A)$.

Remark 3.9. For any subset A of a BGT-space X, $A \subseteq \mu_{mn}S_p c_{\mu_{mn}}(A)$.

4 $\mu_{mn}S_p$-Interior and $\mu_{mn}S_p$-Closure of a Set

In this section, we present some results involving $\mu_{mn}S_p$-interior and $\mu_{mn}S_p$-closure of a set in the BGT-space. First, consider the following remark:

Remark 4.1. Let (X, μ_m, μ_n) be a BGT-space and $A \subseteq X$. Then
(i) $\mu_{mn}S_p c_{\mu_{mn}}(A) = X\backslash \mu_{mn}S_p i_{\mu_{mn}}(X\backslash A)$;
(ii) $\mu_{mn}S_p i_{\mu_{mn}}(A) = X\backslash \mu_{mn}S_p c_{\mu_{mn}}(X\backslash A)$.
(iii) A is μ_{mn}-semiopen and μ_{mn}-preclosed if and only if $A = c_{\mu_m}(i_{\mu_n}(A))$.
(iv) If $A = c_{\mu_{mn}}(i_{\mu_{mn}}(A))$, then A is $\mu_{mn}S_p$-open.

The converse of Remark 4.1 (iv) need not be true. Consider the same BGT-space X in Remark 3.4. Observe that the set $A = \{a, b\}$ is $\mu_{mn}S_p$-open and $c_{\mu_{mn}}(i_{\mu_{mn}}(A)) = \{a, b, c\}$ which means $A \neq c_{\mu_{mn}}(i_{\mu_{mn}}(A))$.

Lemma 4.2. *Arbitrary union of μ_{mn}-semiopen sets is μ_{mn}-semiopen.*

Proof.
Let $\{M_i : i \in I\}$ be a collection of μ_{mn}-semiopen sets in a BGT-space X. Then $M_i \subseteq c_{\mu_m}(i_{\mu_n}(M_i))$ for all i. Thus

$$\begin{aligned}\bigcup_{i\in I} M_i &\subseteq \bigcup_{i\in I} c_{\mu_m}(i_{\mu_n}(M_i)) \\ &\subseteq c_{\mu_m}\left(\bigcup_{i\in I} i_{\mu_n}(M_i)\right) \\ &\subseteq c_{\mu_m}\left(i_{\mu_n}(\bigcup_{i\in I} M_i)\right).\end{aligned}$$

Therefore, $\bigcup_{i\in I} M_i$ is μ_{mn}-semiopen. □

Theorem 4.3. *The collection of all $\mu_{mn}S_p$-open sets in X forms a BGT on X.*

Proof.
Let $C = \{M_i : M_i \text{ is } \mu S_p\text{-open}, i \in I\}$. Clearly, $\varnothing$ is $\mu_{mn}S_p$-open. Since M_i is $\mu_{mn}S_p$-open for all $i \in I$, M_i is μ_{mn}-semiopen for all i. By Lemma 4.2, $\cup_{i\in I} M_i$ is μ_{mn}-semiopen. Let $x \in \cup_{i\in I} M_i$. Then $x \in M_i$ for some $i \in I$. Since M_i is $\mu_{mn}S_p$-open, there exists a μ_m-preclosed set F_i such that $x \in F_i \subseteq M_i$. This implies that $x \in F_i \subseteq \cup_{i\in I} M_i$. Therefore, $\cup_{i\in I} M_i$ is $\mu_{mn}S_p$-open. It follows that C forms a BGT on X. □

Corollary 4.4. *The intersection of all $\mu_{mn}S_p$-closed sets is $\mu_{mn}S_p$-closed.*

Proof.
Let F_i be $\mu_{mn}S_p$-closed sets for each $i \in I$. Then $X\backslash F_i$ is $\mu_{mn}S_p$-open for each i. By Theorem 4.3, $\cup_{i\in I}(X\backslash F_i) = X\backslash(\cap_{i\in I} F_i)$ is $\mu_{mn}S_p$-open. Therefore, $\cap_{i\in I} F_i$ is a $\mu_{mn}S_p$-closed set. □

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