International Journal of Mathematics and
Computer Science, **19**(2024), no. 4, 1421–1425

Power Dominator Equitable Coloring of Graphs

Sharmila Mary Arul, Diana Grace Thomas, Swati Kharub

Department of Mathematics
Saveetha School of Engineering
Saveetha Institute of Medical and Technical Sciences
Saveetha University
Chennai, India

email: sharmilamaryarul.sse@saveetha.com

(Received April 15, 2024, Accepted May 16, 2024,
Published June 1, 2024)

Abstract

The concept of power dominator equitable coloring is proposed in this paper as a novel graph coloring technique. A power dominator coloring of a graph G is a proper coloring in which each vertex power dominates every vertex of some color class. The power dominator equitable coloring is a proper k-coloring of G, in which every vertex of G power dominates all vertices of some color class $C_1, C_2, C_3, \ldots$ or C_k such that the difference in size between any two color classes is at most 1. The minimum number of colors in a power dominator equitable coloring set of vertices in a graph G is called the power dominator equitable chromatic number of G, denoted by $\chi_{pde}(G)$. In this paper we have obtained χ_{pde} for some special classes of graphs.

1 Introduction

In this paper, we introduce the concept of power dominator equitable coloring, a variant of proper vertex coloring in graphs. This coloring ensures that

Key words and phrases: Color class, power dominator coloring, equitable coloring, graphs.
AMS (MOS) Subject Classifications: 05C15, 05C85.
ISSN 1814-0432, 2024, http://ijmcs.future-in-tech.net

each vertex power dominates every vertex in some color class, maintaining a balance in the number of vertices across color classes. The application of this concept is significant in computer networks, where high-power nodes dominate others, ensuring network stability and resilience to failures.

Power domination, introduced by Cockayne et al. [1] in 1998 was initially motivated by monitoring electric power systems. Haynes et al. [2] further explored this concept in 2003. The application of power domination is notable in modeling trash collection routes, aiming to minimize scheduling conflicts. Equitable coloring, proposed by Mayer [4] in 1973, ensures a balanced distribution of allocating workdays to routes on each day [3]. Combining power dominator coloring and equitable coloring, we introduce the concept of power dominator equitable coloring.

2 Power Dominator Equitable Coloring

We now give a formal definition of Power Dominator Equitable Coloring of a graph. For a vertex v in G, let $N(v)$ denote its open neighbourhood.

Definition 2.1. *Let $v \in V(G)$, the monitoring set $M(v)$ is constructed as follows:*
Step 1: $M(v) = N(v) \cup \{v\}$
Step 2: Add a vertex u in $V(G) \setminus M(v)$ to $M(v)$ whenever u has a neighbor $w \in M(v)$ such that all the neighbors of w other than u are already in $M(v)$.
Step 3: Repeat Step 2 till no more vertices be added to $M(v)$.
We say that the vertex v power dominates the vertices in $M(v)$.

Definition 2.2. *The power dominator equitable coloring of a graph G is a proper k-coloring of vertices of G with color classes $C_1, C_2, \ldots, C_k$ such that (i) each vertex of G power dominates all the vertices of at least one color class $C_1, C_2, \ldots C_k$ and (ii) $||C_i| - |C_j|| \leq 1, 1 \leq i, j \leq k$. The minimum number of colors needed for this coloring is termed as the power dominator equitable chromatic number and is denoted by $\chi_{pde}(G)$.*

In this paper χ_{pde} has been determined for Path Graph P_n, Cycle Graph C_n, Complete Graph K_n, Wheel Graph W_n, Star Graph S_n, Bistar Graph $ST(m,n)$, Helm Graph H_n, Sunflower Graph SF_n, Book Graph B_n, Lollipop Graph $L_{n,m}$, Tadpole Graph $T_{m,n}$, Spider Graph S_n and Pan Graph PN_n. We refer to [5, 6] for the definitions of these graphs.

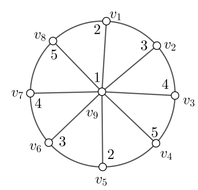

Figure 1: Wheel Graphs with $\chi_{pde}(W_9) = 5$

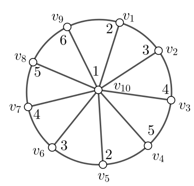

Figure 2: Wheel Graphs with $\chi_{pde}(W_{10}) = 6$

Theorem 2.3. *We have the following results:*

(i) $\chi_{pde}\left(P_n\right) = 2$, *for* $n \geq 2$.

(ii)

$$\chi_{pde}(C_n) = \begin{cases} 3 & \text{if } n \text{ is odd, for } n \geq 3, \\ 2 & \text{if } n \text{ is even, for } n \geq 4. \end{cases}$$

(iii) $\chi_{pde}\left(K_n\right) = n$, *for* $n \geq 3$.

$$\chi_{pde}(W_n) = \begin{cases} 4 & \text{for } n = 4, \\ \lfloor \frac{n}{2} \rfloor + 1 & \text{for } n \geq 5. \end{cases}$$

(v)

$$\chi_{pde}(S_n) = \begin{cases} \lfloor \frac{n}{2} \rfloor & \text{for } n \text{ odd }, n \geq 5, \\ \frac{n}{2} + 1 & \text{for } n \text{ even }, n \geq 4. \end{cases}$$

(vi) $\chi_{pde}(ST(m,n)) = m + n - 2$, *for* $m, n \geq 3$.
(vii) $\chi_{pde}\left(H_n\right) = n + \left\lceil \frac{n+1}{2} \right\rceil$, *for* $n \geq 3$.
(viii) $\chi_{pde}\left(SF_n\right) = 2$, *for* $n \geq 2$.
(ix) $\chi_{pde}\left(B_n\right) = n + 2$, *for* $n \geq 3$.
(x) $\chi_{pde}\left(L_{n,m}\right) = n$, *for* $n \geq 2, m \geq 1$.
(xi) $\chi_{pde}\left(T_{m,n}\right) = 3$, *for* $n \geq 1, m \geq 3$.

(xii) $\chi_{pde}(S_n) = 3$, *for* $n \geq 5$.
(xiii) $\chi_{pde}(PN_n) = 2$, *for* $n \geq 3$.

Proof. (i) Let G be a Path graph P_n with vertices $v_1, v_2, \ldots, v_n$. By assigning colors 1 and 2 alternately to the vertices of G. Each vertex v_i (odd index) power dominates the color class 2, and each vertex v_i (even index) power dominates the color class 1. The result follows.
(ii) Let G be a Cycle graph C_n with vertices $v_1, v_2, \ldots, v_n$. Assign colors 1 and 2 alternately to the vertices of G. Each vertex v_i, i odd, power dominates the color class 2, and each vertex v_i, i even, power dominates the color class 1. Hence, the proof obtains.
(iii) Let G be a Complete graph K_n with vertices $v_1, v_2, \ldots, v_n$. Assign distinct colors $1, 2, \ldots, n$ to the vertices of G. Each vertex v_i power dominates its own color class, ensuring a power dominator equitable coloring with $\chi_{pde}(K_n) = n$.
(iv) Let G be a Wheel Graph W_n with $n \geq 4$. The vertices of W_n consist of a cycle with $n-1$ rim vertices labeled $v_1, v_2, \ldots, v_{n-1}$ in the clockwise sense and a central vertex (hub) denoted as v_n. Then v_i power dominates $v_1, v_2, \ldots, v_n, \forall i, 1 \leq i \leq n$. When $n = 4$, assign color 1 to the hub and colors 2,3,4 to vertices v_1, v_2, v_3 respectively. This ensures that each vertex power dominates its own color class. Hence, $\chi_{pde}(W_n) = 4$. When $n \geq 5$, assign color 1 to the hub and to satisfy equitable coloring pair the remaining non adjacent rim vertices $v_1, v_2, \ldots, v_{n-1}$ in different color classes such that each color class has at most two vertices. When n is odd, assign color $i+1$ to the vertices v_i and $v_{i+\frac{n-1}{2}}$, $1 \leq i \leq \frac{n-1}{2}$, inducing a color class of cardinality 2. When n is even, assign color $i+1$ to the vertices v_i and $v_{i+\frac{n-2}{2}}$, $1 \leq i \leq \frac{n-2}{2}$, inducing a color class of cardinality 2 and color $\frac{n}{2}+1$ to vertex v_{n-1} inducing a color class of cardinality 1. Thus, $\chi_{pde}(W_n) = \lfloor \frac{n}{2} \rfloor + 1$. See Figure 1. This assignment ensures that each vertex power dominates its own colors, and the hub vertex power dominates all the rim vertices. Hence, the proof obtains. Similar arguments lead to the rest of the results.

3 Conclusion

It would be an interesting line of research to explore the power dominator equitable coloring problem in hypercubes and butterfly networks.

References

[1] E.J. Cockayne, S.T. Hedetniemi, Towards a theory of domination in graphs, Networks, **7,** (1977), 247–261.

[2] T.W. Haynes, S.M. Hedetniemi, S.T. Hedetniemi, M.A. Henning, Domination in graphs applied to electric power networks, SIAM J. Discrete Math., **15,** no. 4, (2002), 519–529.

[3] K.S. Kumar, D.N. Gnanamalar, K.G. Subramanian, "Graphs and power dominator colorings", Ann. Pure Appl. Math., **11,** no. 2, (2016), 67–71.

[4] W. Meyer, "Equitable coloring", Amer. Math. Monthly, **80,** (1973), 920–922.

[5] A. Brandstadt, V.B. Le, J.P. Spinrad, "Graph classes: a survey," SIAM, 1999.

[6] Joseph A. Gallian, "A Dynamic Survey of Graph Labeling," Electronic Journal of Combinatorics, Dynamic Surveys, DS6, (2018).