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Dominator Sum Coloring Algorithm for Comb-Double Comb Graph

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Abstract

A dominator coloring of a proper coloring of a graph G where each vertex dominates all the vertices of at least one color class (possibly its own color class) is denoted by the dominator chromatic number is $\chi_d(G)$, and chromatic sum Σ(G), while minimizing the sum of labels (natural numbers) assigned to the vertices. The dominator sum chromatic number, denoted as $\chi_{ds}(G)$, represents the minimum number of color classes required to achieve a dominator sum coloring of G while minimizing the sum of labels. In this paper, we prove that the dominator sum coloring algorithm for comb graph and double comb graph.

1 Introduction

Coloring concepts in graph theory have real-world applications in scheduling and resource allocation. Graph coloring involves the assignment of colors

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to vertices while ensuring that adjacent vertices have distinct colors. The concept of dominator was introduced by Ralucca Michelle Gera in 2006 [5]. Dominator colorings in graphs was introduced by Cockayne, S.M. Hedetniemi, and S.T. Hedetniemi[1, 2, 3]. in their seminal work on the dominator number of a graph Hedetniemi, McRae, Laskar, and Wallis have further explored and refined dominator coloring[5, 8]. We have introduced Dominator Sum Coloring of graphs. Practical applications of dominator sum coloring are diverse and impactful, with potential uses in network optimization, scheduling algorithms, resource allocation,[7], and task assignment problems. By balancing dominator properties with label minimization[6, 7, 9], The efficiency it enhances in optimization problems in diverse fields.

2 Main results

Definition 2.1. *Proper Coloring [4]*
A proper coloring of a graph $G = (V(G), E(G))$ is an assignment of colors to the vertices of the graph, such that any two adjacent vertices receive different colors. The chromatic number is the minimum number of colors needed in a proper coloring of the graph.

Definition 2.2. *Dominator Coloring [1]*
A dominator coloring (DC) of a graph G is a proper vertex coloring of G such that each vertex dominates some color class or else lies alone in its color class. A dominator chromatic number $\chi_d(G)$ is the minimum cardinality among all DCs of G.

We introduce dominator sum coloring as follows

Definition 2.3. *Dominator sum coloring is a combination of dominator coloring and sum coloring. It is a proper coloring of a graph G where each vertex dominates all the vertices of at least one color class (possibly its own color class) while minimizing the sum of labels (natural numbers) assigned to the vertices. The dominator sum chromatic number, denoted as $chi_{ds}(G)$, represents the minimum number of color classes required to achieve a dominator sum coloring of G while minimizing the sum of labels.*

Definition 2.4. *Chromatic Sum*
A proper vertex coloring of G, using natural numbers, such that the total sum of the colors of vertices is minimized among all dominator colorings of G is called the chromatic sum of G, denoted by $\Sigma(G)$.

Definition 2.5. *An r-regular caterpillar $c(n, 1)$ is a graph obtained by attaching r pendent edges at each vertex of a path P_n. A 1-regular caterpillar $c(n, 1)$ is called a comb, and a 2-regular caterpillar $c(n, 2)$ is called a bi-comb.*

Theorem 2.6. *Let $c(n, 1)$ be the Comb graph on $2n$ vertices. Then $\chi_{ds}c(n,1) = n + 1$, and $\Sigma_{ds}c(n, 1) = \frac{n^2+5n}{2}$.*

Proof. The comb $c(n, 1)$ has n spine vertices and n leaf vertices. Label the leaf vertices as 1 and the spine vertices as $2, 3, \ldots, n+1$ from left to right. Each spine vertex color dominates itself. Each leaf vertex color dominates the spine vertices to which it is attached. $\chi_d = n + 1$, $\chi_{ds} = n + 1$, and $\sum \chi_{ds} = \frac{n^2+5n}{2}$.

Theorem 2.7. *Let G be* Bi - *comb graph on $3n$ vertices. Then $\chi_{ds}c(n, 2) = n + 1$ and $\sum_{ds} = n^2 + 5n$*

Proof. The comb $c(n, 2)$ has n spine vertices and $2n$ leaf vertices. Label the leaf vertices as 1 and the spine vertices as $2, 3, \ldots, n+1$ from left to right. Each spine vertex color dominates itself. Each leaf vertex color dominates the spine vertices to which it is attached. Hence $\chi_{ds} = n+1$, and $\sum_{ds} = n^2+5n$.

3 Conclusion

Dominator sum coloring, applied to standard graphs like Comb and Double-Comb graphs, determines chromatic sum and dominator sum chromatic number. It minimizes label sums while ensuring dominator conditions, offering insights into graph coloring theory. These findings enrich algorithm design and optimization in diverse graph-related problems. Future research may extend dominator sum coloring to other graphs for practical applications.

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