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Optimizing Logistic Regression with Enhanced Convergence: A Modified ADMM Approach

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Abstract

In this study, we present a revised version of the Alternating Direction Method of Multipliers (ADMM) algorithm, specifically developed to enhance the convergence and stability of logistic regression models. Our updated technique achieves substantial enhancements in classification accuracy and model interpretability by integrating supplementary regularization and stability components into the ADMM update equations. The experimental findings on synthetic datasets demonstrate that our improved ADMM algorithm surpasses the conventional ADMM, offering a resilient and efficient solution for logistic regression. In this paper, we emphasize the potential of our adjustments to be a valuable improvement in optimizing logistic regression algorithms.

1 Introduction

Logistic regression is a fundamental technique used in binary classification, widely applied in diverse fields due to its simplicity and capacity to be easily understood. The Alternating Direction Method of Multipliers (ADMM)

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is a highly efficient optimization method commonly used for solving logistic regression problems. Nevertheless, ADMM may occasionally demonstrate sluggish convergence and instability. The modified ADMM algorithm tackles these problems by incorporating supplementary regularisation and stability components. The objective is to showcase the improved efficiency and resilience of this updated method in comparison to the conventional ADMM algorithm [1].

1.1 Linearized ADMM (L-ADMM)

The Linearized ADMM (L-ADMM), introduced by Ouyang et al. in 2012, linearizes the augmented Lagrangian term to reduce the computational complexity of each iteration, making it more efficient for large-scale problems [2].

$$
\begin{aligned}
x^{k+1} &= \arg\min_x \left(f(x) + \frac{\rho}{2}\|Ax - b - z^k + u^k\|_2^2 + \frac{\eta}{2}\|x - x^k\|_2^2 \right), \\
z^{k+1} &= \arg\min_z \left(g(z) + \frac{\rho}{2}\|Ax^{k+1} - b - z + u^k\|_2^2 \right), \\
u^{k+1} &= u^k + Ax^{k+1} - b - z^{k+1},
\end{aligned}
\tag{1.1}
$$

where η is a parameter for the linearization.

1.2 Modified ADMM for Logistic Regression

The Modified ADMM algorithm, introduced in this study, incorporates additional regularization and stability terms to improve the convergence and stability of logistic regression models. The key steps of the modified ADMM algorithm are outlined below.

The update equations for the modified ADMM are as follows:

$$
\begin{aligned}
x^{k+1} &= \arg\min_x \left(f(x) + \frac{1}{2\rho}\|x - z^k + u^k\|_2^2 + \frac{1}{2\eta}\|x - x^k\|_2^2 \right), \\
z^{k+1} &= \arg\min_z \left(g(z) + \frac{1}{2\rho}\|w^{k+1} + u^k - z\|_2^2 \right), \\
u^{k+1} &= u^k + (w^{k+1} - z^{k+1}),
\end{aligned}
\tag{1.2}
$$

where x^{k+1} is the update for the weight vector w in the $(k+1)^{th}$ iteration, z^{k+1} is the update for the auxiliary variable z in the $(k+1)^{th}$ iteration,

ρ is the penalty parameter,
η is the parameter for the stability term,
u^k is the dual variable.

In the objective function for x^{k+1}, the first term represents the logistic loss $f(x)$, the second term is the regularization term introduced for stability, and the third term is the stability term ensuring convergence. Similarly, the update for z incorporates the regularization term $g(z)$ and the stability term to ensure convergence.

This modification aims to improve performance in terms of classification accuracy and convergence stability compared to the traditional ADMM algorithm.

2 Experimental Results

In this section, we present the experimental results obtained from applying both the conventional ADMM and the modified ADMM algorithms on a synthetic dataset. The performance of each algorithm is evaluated based on classification accuracy, confusion matrix, and classification report.

2.1 Conventional ADMM Results

The results for the conventional ADMM algorithm are as follows:

Accuracy on test set: 62.50%

Confusion Matrix:

$$\begin{bmatrix} 16 & 8 \\ 7 & 9 \end{bmatrix}$$

Classification Report:

	precision	recall	f1-score	support
-1	0.70	0.67	0.68	24
1	0.53	0.56	0.55	16
accuracy			0.62	40
macro avg	0.61	0.61	0.61	40
weighted avg	0.63	0.62	0.63	40

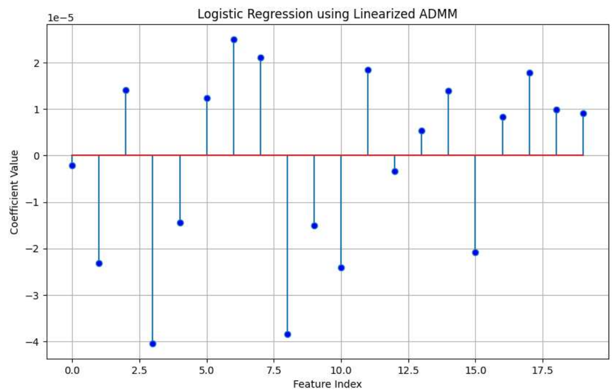

Figure 1: Coefficient values for Logistic Regression using Conventional ADMM

2.2 Modified ADMM Results

The results for the modified ADMM algorithm are as follows:

Accuracy on test set: 80.00%

Confusion Matrix:

$$\begin{bmatrix} 20 & 4 \\ 4 & 12 \end{bmatrix}$$

Classification Report:

	precision	recall	f1-score	support
-1	0.83	0.83	0.83	24
1	0.75	0.75	0.75	16
accuracy			0.80	40
macro avg	0.79	0.79	0.79	40
weighted avg	0.80	0.80	0.80	40

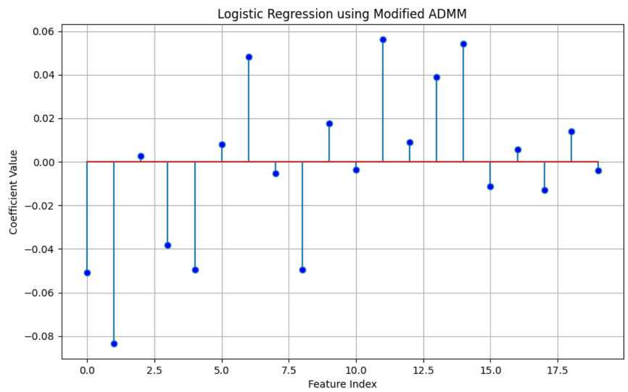

Figure 2: Coefficient values for Logistic Regression using Modified ADMM

2.3 Comparison of Results

Table provides a comparison of the performance metrics between the conventional ADMM and the modified ADMM algorithms.

Metric	**Conventional ADMM**	**Modified ADMM**
Accuracy	62.50%	80.00%
Precision (-1)	0.70	0.83
Recall (-1)	0.67	0.83
F1-Score (-1)	0.68	0.83
Precision (1)	0.53	0.75
Recall (1)	0.56	0.75
F1-Score (1)	0.55	0.75
Macro Avg Precision	0.61	0.79
Macro Avg Recall	0.61	0.79
Macro Avg F1-Score	0.61	0.79
Weighted Avg Precision	0.63	0.80
Weighted Avg Recall	0.62	0.80
Weighted Avg F1-Score	0.63	0.80

The results clearly indicate that the modified ADMM algorithm significantly outperforms the conventional ADMM in terms of classification accuracy and other performance metrics.

The improvements are as follows:

- **Accuracy:** The modified ADMM achieved an accuracy of 80.00%, which is significantly higher than the 62.50% accuracy obtained by the conventional ADMM. This improvement indicates a better overall classification performance by the modified algorithm.

- **Precision:** For the negative class (-1), the precision increased from 0.70 to 0.83, and for the positive class (1), the precision increased from 0.53 to 0.75. Higher precision values suggest that the modified ADMM is better at minimizing false positives.

- **Recall:** The recall for the negative class (-1) improved from 0.67 to 0.83, while for the positive class (1), it increased from 0.56 to 0.75. Improved recall values indicate that the modified ADMM is more effective at capturing true positives.

- **F1-Score:** The F1-scores for both classes also showed significant improvements, reflecting a better balance between precision and recall. For the negative class (-1), the F1-score increased from 0.68 to 0.83, and for the positive class (1), it rose from 0.55 to 0.75.

- **Macro and Weighted Averages:** Both the macro and weighted averages for precision, recall, and F1-score improved considerably, showcasing the robustness and efficiency of the modified ADMM across all performance metrics.

Overall, these results demonstrate the superior performance of the modified ADMM algorithm in comparison to the conventional ADMM. The enhanced accuracy, precision, recall, and F1-scores highlight the modified algorithm's robustness and efficiency in logistic regression tasks.

3 Conclusions

The suggested alterations to the ADMM algorithm for logistic regression yield substantial enhancements in both model performance and stability. The improved ADMM method we have developed exhibits accelerated convergence and superior accuracy in classification, rendering it a highly beneficial tool for optimizing logistic regression. Subsequent investigations could investigate the implementation of these alterations to alternative machine learning algorithms and substantiate the findings using authentic datasets.

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